

An Analytic Approach to Reputation Ranking of Participants in Online Transactions

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Abstract

Agents from a community interact in pairwise transactions across discrete time. Each agent reports its evaluation of another agent with which it has just had a transaction to a central system. This system uses these time-sequences of experience evaluations to infer how much the agents trust each another. Our paper proposes rationality assumptions (also called axioms or constraints) that such inferences must obey, and proceeds to derive theorems implied by these assumptions. A basic representation theorem is proved. The system also uses these pairwise cross-agent trustworthiness to compute a reputation rank for each agent. Moreover, it provides with each reputation rank an estimate of the reliability, which we call weight of evidence. This paper is different from much of the current work in that it examines how a central system which computes trustworthiness, reputation and weight of evidence is constrained by such rationality postulates.

1 Introduction

With the rapid increase of commerce and other online transactions, it has become advantageous to have methods for evaluation of the behavior of agents involved. The notions that address how agents assess other agents, and how overall reputations arise, derive from cognitive social science. The primary reference for this background is the foundational work [1]. Some well-known and widely used systems already give some data to help assess the reputation of an agent. For example, the *eBay* makes available for each agent the percentage of positive evaluations it receives from other agents, called the “*Feedback*”. The report [6] has a comprehensive survey of many alternative approaches to representing, assessing, computing and updating agent

trustworthiness and reputation. This paper is different in spirit of much of the ongoing work on reputation of which [5] is a good example. In [5] the main method for evaluating performance of different approaches is by simulation. Our paper however sets out postulates that are intuitively appealing, and from them we derive representation theorems.

Assume that N agents $\{A_i : 1 \leq i \leq N\}$ are involved in some pairwise transactions over discrete time instants. We identify time instants with natural numbers and that the “granularity” of time intervals is sufficiently fine so that any two agents at any instant can be involved in at most one transaction. For an agent A_i , its assessment of another agent A_j for a transaction happened at k is its *experience evaluation*, $e_{ij}(k)$. Over time up to n , agent A_i has produced an *experience sequence* which we identify with the *partial function* $\vec{s}_{ij} = \{(k, e_{ij}(k)) : k \in D(\vec{s}_{ij})\}$. The domain $D(\vec{s}_{ij})$ is the collection of time instants $k \leq n$ when agent A_i was engaged in a transaction with A_j . If there is no confusion, we drop the index i, j for brevity. For every $k \in D(\vec{s})$, $\vec{s}[k] = e(k)$. This sequence is the base of A_i ’s judgement of A_j ’s *trustworthiness* which can be seen as an aggregation over time of all individual past experiences. We call this *trust rank*, $T(\vec{s})$.

For an agent A_j , its trust ranks by all other agents in the community can be further aggregated to give a *community-wide judgement* on its trustworthiness. We call this A_j ’s *reputation*, $\rho_j(n)$, which is valid at time n . Having a reputation based on just a few transactions long time ago is clearly different from having the same based on a large number of recent transactions. To address this issue, we introduce the notion of *community evidence*, $W_j(n)$, for each $\rho_j(n)$.

In our approach, these two levels of aggregation are performed by a central mediating *agency* to minimize bias in the aggregation process. In addition to being more robust against manipulative reporting, this system also maintains the privacy of the evaluations of individual transactions.

Fair conditions, also called rationality assumptions, that constrain how an experience sequence \vec{s}_{ij} is related to the consequent trustworthiness assessment have been investigated in [4] and [2]. We provide a rigorous analysis of the problem of ranking the trustworthiness and reputation of agents involved in transactions, and in assessing reliability of such a rank, based on our rationality assumptions. While these persuasive assumptions are essential to our analysis, much of our technical development can be modified for somewhat different setups that do not meet all of our assumptions.

2 Aggregation in Time

We will assume that an experience evaluation $e(k)$ is a real number in a fixed range $[0, M]$, $M > 0$. If $e(k) = M$, this means A_i regards the quality of the transaction with A_j at moment k is the best, while a report of 0 means the worst. Now, let us list axioms which delineate the “rationality” of trust rank T .

2.1 Axioms for Aggregation in Time

(T1) Shift Invariance. Let \vec{s}^+ be the forward unit shift of the sequence \vec{s} , i.e., let $\vec{s}^+ = \langle (k+1, e(k)) : k \in D(\vec{s}) \rangle$; then $T(\vec{s}^+) = T(\vec{s})$.

(T2) Time Averaging. Let \vec{s} be any sequence of experiences; then $\min(\vec{s}) \leq T(\vec{s}) \leq \max(\vec{s})$.

(T3) Consistency. Let \vec{s}_1 and \vec{s}_2 be two experience sequences such that $D(\vec{s}_1) \cap D(\vec{s}_2) = \emptyset$ and $T(\vec{s}_1) = T(\vec{s}_2)$; then $T(\vec{s}_1) = T(\vec{s}_1 \cup \vec{s}_2) = T(\vec{s}_2)$.

(T4) Discounting. Assume that $0 \leq e < E \leq M$ and let $\vec{s}_1 = \{(1, e), (2, E)\}$ and $\vec{s}_2 = \{(1, E), (2, e)\}$; then $T(\vec{s}_1) \geq T(\vec{s}_2)$.

We feel that every reasonable trust rank function must satisfy the above axioms. Intuitive justification of the next axiom given below is perhaps somewhat less than universal; however, it greatly simplifies technical matters, without imposing any undue restrictions.

(T5) Linearity. Let α, β be real numbers, and \vec{s}_1, \vec{s}_2 be two sequences such that $D(\vec{s}_1) = D(\vec{s}_2) = D$ and $0 \leq \alpha \vec{s}_1[k] + \beta \vec{s}_2[k] \leq M$ for all $k \in D$. Further, the sequence \vec{s} is defined on D with $\vec{s}[k] = \alpha \vec{s}_1[k] + \beta \vec{s}_2[k]$ for all $k \in D$. Then $T(\vec{s}) = \alpha T(\vec{s}_1) + \beta T(\vec{s}_2)$.

The next lemma reveals a form for trust rank functions analogous to the response of a linear system.

Lemma 2.1 *Let $n \in \mathbb{N}$ and $I_n = \{i : 1 \leq i \leq n\}$. Further, let $D \subseteq I_n$ and $\{1, n\} \subseteq D$. For a sequence \vec{s} with $D(\vec{s}) = D$, if the trust rank function T satisfies **T1** through **T5**, there exist positive numbers w_k^D such that*

$$\sum_{k \in D} w_k^D = 1 \quad (1)$$

$$T(\vec{s}) = \sum_{k \in D} w_k^D \vec{s}[k]. \quad (2)$$

Proof. For each $k \in D$, we define the experience sequence \vec{s}_k^D such that $D(\vec{s}_k^D) = D$; $\vec{s}_k^D[k] = 1$ and $\vec{s}_k^D[m] = 0$ for all $m \in D, m \neq k$. Then we have $\vec{s} = \sum_{k \in D} \vec{s}[k] \vec{s}_k^D$. Since T is linear, $T(\vec{s}) = \sum_{k \in D} \vec{s}[k] T(\vec{s}_k^D)$. Consider the sequence $\vec{1}_D$ such that for all $i \in D$, $\vec{1}_D(i) = 1$. $T(\vec{1}_D) = \sum_{k \in D} T(\vec{s}_k^D)$. By Averaging Axiom, $T(\vec{1}_D) = 1$; thus, $\sum_{k \in D} T(\vec{s}_k^D) = 1$, and for all $k \in D$ we can take $w_k^D = T(\vec{s}_k^D)$. \square

Corollary 2.2 (Uniform Continuity) *For every δ and every two experience sequences \vec{s}_1, \vec{s}_2 with $D(\vec{s}_1) = D(\vec{s}_2) = D$ and $|\vec{s}_1[k] - \vec{s}_2[k]| < \delta$ for all $k \in D$, then also $|T(\vec{s}_1) - T(\vec{s}_2)| < \delta$.*

Corollary 2.3 (Point-wise monotonicity) *Let \vec{s}_1, \vec{s}_2 be two experience sequences with $D(\vec{s}_1) = D(\vec{s}_2) = D$. If $\vec{s}_1[k] \geq \vec{s}_2[k]$ for all $k \in D$, then $T(\vec{s}_1) \geq T(\vec{s}_2)$.*

Theorem 2.4 (Representation Theorem) *Let T satisfy the axioms; then there exists $q \geq 1$ such that for all \vec{s} the following canonical representation of $T(\vec{s})$ holds:*

$$T(\vec{s}) = \sum_{k \in D(\vec{s})} \frac{q^{k-1}}{\sum_{l \in D(\vec{s})} q^{l-1}} e(k). \quad (3)$$

Proof. Note that, by Lemma 2.1, for some $w_1^{I_2}, w_2^{I_2}$ such that $w_1^{I_2} + w_2^{I_2} = 1$ we have $T(\{(1, 1), (2, 0)\}) = w_1^{I_2}$ and $T(\{(1, 0), (2, 1)\}) = w_2^{I_2}$. By **T4**, $w_1^{I_2} \leq w_2^{I_2}$. Thus, $w_1^{I_2} \leq 1/2$ and so for some $q \geq 1$, $T(\{(1, 1), (2, 0)\}) = 1/(1+q)$ and consequently $T(\{(1, 0), (2, 1)\}) = w_2^{I_2} = 1 - w_1^{I_2} = q/(1+q)$.

We first prove the statement for all D such that $D = I_n = \{1, \dots, n\}$ for some $n \geq 3$, and for q as above. Using Lemma 2.1, $T(\vec{s}) = \sum_{k=0}^n w_k^n \vec{s}[k]$. Fix $m < n$ and consider a set of 3 experience sequences $\vec{s}^{mm'}$, $\vec{s}^{\bullet mm'}$ and $\vec{s}^{*mm'}$ with $D(\vec{s}^{mm'}) = I_n$, $D(\vec{s}^{\bullet mm'}) = \{m, m+1\}$, $D(\vec{s}^{*mm'}) = I_n \setminus D(\vec{s}^{\bullet mm'})$. We define $\vec{s}^{mm'}$ as follows:

$$\vec{s}^{mm'}[k] = \begin{cases} 1 & \text{if } k = m \\ 0 & \text{if } k = m+1 \\ 1/(1+q) & \text{otherwise;} \end{cases}$$

while $\vec{s}^{\bullet mm'} = \{(m, 1), (m+1, 0)\}$ and $\vec{s}^{*mm'}[k] = 1/(1+q)$ for all $k \in D(\vec{s}^{*mm'})$. Then by **T1**, $T(\vec{s}^{\bullet mm'}) = T(\{(1, 1), (2, 0)\}) = 1/(1+q)$. By **T2**, $T(\vec{s}^{*mm'}) = 1/(1+q)$. Thus, by **T3**, $T(\vec{s}^{mm'}) = T(\vec{s}^{\bullet mm'} \cup \vec{s}^{*mm'}) = 1/(1+q)$. Since by Lemma 2.1 $\sum_{k \in I_n} w_k^n = 1$, we get $T(\vec{s}^{mm'}) = w_m^n + (1 - w_m^n - w_{m+1}^n)/(1+q) = 1/(1+q)$, which implies $w_{m+1}^n = w_m^n q$. Consequently, for all $k \in I_n$, $w_k^n = q^{k-1}/(1 + \dots + q^{n-1}) = q^{k-1}(q-1)/(q^n-1)$.

Consider now another set of sequences \vec{s}^{1n} , $\vec{s}^{\bullet 1n}$ and \vec{s}^{*1n} with $D(\vec{s}^{1n}) = I_n$, $D(\vec{s}^{\bullet 1n}) = \{1, n\}$, $D(\vec{s}^{*1n}) = I_n \setminus D(\vec{s}^{\bullet 1n})$. Let this time $\vec{s}^{\bullet 1n} = \{(1, 1), (n, 0)\}$; then

there exists $r > 0$ such that $T(\vec{s}^{\bullet 1n}) = 1/(1+r)$. For all $k \in D(\vec{s}^{\star 1n})$, $\vec{s}^{\star 1n}[k] = 1/(1+r)$. We define \vec{s}^{1n} as follow:

$$\vec{s}^{1n}[k] = \begin{cases} 1 & \text{if } k = 1 \\ 0 & \text{if } k = n \\ 1/(1+r) & \text{otherwise;} \end{cases}$$

Then again by **T2** and **T3**, $T(\vec{s}^{1n}) = T(\vec{s}^{\bullet 1n}) = T(\vec{s}^{\star 1n}) = 1/(1+r)$. This implies that

$$T(\vec{s}^{1n}) = \frac{1}{\sum_{k=1}^n q^{k-1}} + \frac{1}{1+r} \frac{\sum_{k=2}^{n-1} q^{k-1}}{\sum_{k=1}^n q^{k-1}} = \frac{1}{1+r},$$

which is easily seen to yield $r = q^{n-1}$. This means that for every \vec{s} such that $D(\vec{s}) = \{1, n\}$ we have

$$T(\vec{s}) = \frac{\vec{s}[1]}{1+q^{n-1}} + \frac{\vec{s}[n] q^{n-1}}{1+q^{n-1}}.$$

Finally we consider an arbitrary $D \subseteq \{1, \dots, n\}$ and an arbitrary \vec{s} such that $D(\vec{s}) = D$. Then $T(\vec{s}) = \sum_{k \in D} w_k^D \vec{s}[k]$. To complete the proof of the theorem it is enough to show that if $l, m \in D$ and $l < m$, then $w_m^D/w_l^D = q^{m-l}$. Consider a similar set of sequences \vec{s}^{lm} , $\vec{s}^{\bullet lm}$ and $\vec{s}^{\star lm}$. Let $\vec{s}^{\bullet lm} = \{(l, 1), (m, 0)\}$. By **T1**, $T(\vec{s}^{\bullet lm}) = T(\{(1, 1), (m-l+1, 0)\}) = 1/(1+q^{m-l})$. Again, we set $\vec{s}^{lm}[l] = 1$, $\vec{s}^{lm}[m] = 0$ and $\vec{s}^{lm}[k] = 1/(1+q^{m-l})$ for all $k \in D \setminus \{l, m\}$. Likewise, $\vec{s}^{\star lm}[k] = 1/(1+q^{m-l})$ for all $k \in D \setminus \{l, m\}$. Once more, we have $T(\vec{s}^{lm}) = T(\vec{s}^{\bullet lm} \cup \vec{s}^{\star lm}) = T(\vec{s}^{\star lm}) = T(\vec{s}^{\bullet lm})$, i.e.,

$$\begin{aligned} T(\vec{s}^{lm}) &= \frac{w_l^D}{\sum_{k \in D} w_k^D} + \frac{\left(\sum_{k \in D} w_k^D\right) - w_l^D - w_m^D}{\sum_{k \in D} w_k^D} \frac{1}{1+q^{m-l}} \\ &= \frac{1}{1+q^{m-l}} \end{aligned}$$

which implies $w_l^D q^{m-l} = w_m^D$. \square

Corollary 2.5 Let $\vec{s}_0, \vec{s}_1, \vec{s}_2$ be any three experience sequences such that $D(\vec{s}_0) \neq \emptyset$, $D(\vec{s}_0) \cap D(\vec{s}_1) = \emptyset$, and $D(\vec{s}_1) = D(\vec{s}_2)$. Then:

- $T(\vec{s}_0 \cup \vec{s}_1) < T(\vec{s}_0 \cup \vec{s}_2) \Leftrightarrow T(\vec{s}_1) < T(\vec{s}_2)$ (monotonicity);
- $T(\vec{s}_0) > T(\vec{s}_1) \Rightarrow T(\vec{s}_0) > T(\vec{s}_0 \cup \vec{s}_1) > T(\vec{s}_1)$ (interpolation);
- $|T(\vec{s}_0 \cup \vec{s}_1) - T(\vec{s}_0 \cup \vec{s}_2)| < |T(\vec{s}_1) - T(\vec{s}_2)|$ (amortization).

Theorem 2.6 (Trust Update) Let function $Q(D)$ be defined on finite sets of natural numbers so that $Q(D) =$

$\sum_{i \in D} q^i$; let $n > \max(D(\vec{s}))$ and $\vec{s}' = \vec{s} \cup \{(n, e(n))\}$. Then

$$T(\vec{s}') = \frac{Q(D(\vec{s})) T(\vec{s})}{Q(D(\vec{s})) + q^{n-1}} + \frac{q^{n-1} e(n)}{Q(D(\vec{s})) + q^{n-1}}.$$

Proof. By Theorem 2.4 we have

$$\begin{aligned} T(\vec{s}') &= \frac{\left(\sum_{k \in D(\vec{s})} q^{k-1} \vec{s}[k]\right) + q^{n-1} e(n)}{\sum_{k \in D(\vec{s}')} q^{k-1}} \\ &= \frac{Q(D(\vec{s})) \frac{\sum_{k \in D(\vec{s})} q^{k-1} \vec{s}[k]}{Q(D(\vec{s}))} + q^{n-1} e(n)}{Q(D(\vec{s})) + q^{n-1}} \\ &= \frac{Q(D(\vec{s})) T(\vec{s})}{Q(D(\vec{s})) + q^{n-1}} + \frac{q^{n-1} e(n)}{Q(D(\vec{s})) + q^{n-1}}. \end{aligned}$$

To obtain the new values, $T(\vec{s}')$ and $Q(D(\vec{s}'))$, we only need to keep track of two quantities, $T(\vec{s})$ and $Q(D(\vec{s}))$. \square

As q increases, the dependency of the trust rank on most recent samples also increases, i.e., the value of q controls how fast the trust rank updates. The proper value for q should be chosen close to but larger than one, on the basis of statistical data from the community of agents involved, matching the expected volatility of agents and striking the right balance between importance of longer term performance and likelihood of sudden changes in agent's behavior.

3 Aggregation in Community

To handle the issues about community reputation, we first introduce a notion of the *weight of pairwise evidence*, denoted by $w_{ij}(n)$. Let $\vec{s}_{ij}(n)$ be a sequence up to time n we define

$$w_{ij}(n) = \sum_{k \in D(\vec{s}_{ij}(n))} q^{k-n},$$

where q is the same constant as in (2.4). In essence, we count the number of transactions of A_i with A_j , discounting each transaction by the factor $1/q^{n-k}$ where $k \in D(\vec{s}_{ij}(n))$. In this way, the larger the value of $w_{ij}(n)$ the more significant and reliable the value $T(\vec{s}_{ij})$ is at an instant n . Note that w_{ij} satisfies the recursion:

$$w_{ij}(n+1) = \begin{cases} w_{ij}(n)/q + 1 & \text{if } n+1 \in D(\vec{s}_{ij}(n+1)) \\ w_{ij}(n)/q & \text{otherwise} \end{cases}$$

Thus, w_{ij} can also be evaluated recursively; for $q = 1$ we have $w_{ij} \leq n$, and for $q > 1$, $w_{ij}(n)$ are bounded because $w_{ij}(n) \leq \sum_{k=0}^{\|\vec{s}_{ij}(n)\|-1} q^{-k} \leq \sum_{k=0}^{\infty} q^{-k} = q/(q-1)$.

Let $\Delta_j(n)$ denote the set of indices of all agents with which agent A_j has had transactions up to the instant

n . A_j 's reputation rank, $\rho_j(n)$, should be a form of an average over the community of agents in the sense that $\min\{T(\vec{s}_{ij}(n)) : i \in \Delta_j\} \leq \rho_j(n) \leq \max\{T(\vec{s}_{ij}(n)) : i \in \Delta_j\}$. The influence of a trust assessment $T(\vec{s}_{ij}(n))$ on $\rho_j(n)$ should depend on both the interaction significance, $w_{ij}(n)$, and the assessor's reputation, $\rho_i(n)$. The following formula of weighted average satisfies all the above criteria:

$$\rho_j(n) = \sum_{i \in \Delta_j(n)} \frac{f(w_{ij}(n), \rho_i(n)) T(\vec{s}_{ij}(n))}{\sum_{i \in \Delta_j(n)} f(w_{ij}(n), \rho_i(n))}$$

where $f(w, \rho) : [0, q/(q-1)] \times [1, M] \mapsto \mathbb{R}^+$ is a strictly increasing in both arguments and satisfies $w, \rho \leq f(w, \rho)$.

Note that the above does not solve our problem of assigning reputation rank. To solve this problem we look for a solution of the following system of equations in variables ρ_i , $i \leq N$.

$$\left\{ \sum_{i \in \Delta_j(n)} \frac{f(w_{ij}(n), \rho_i) T(\vec{s}_{ij}(n))}{\sum_{k \in \Delta_j(n)} f(w_{kj}(n), \rho_k)} = \rho_j \right\}_{j \leq N} \quad (4)$$

where $\rho_i(n)$, $i \leq N$ are chosen to be the solutions.

For our numerical experiment using $f(w, \rho) = w^\alpha \rho^\beta$ with positive real numbers α, β depending on discounting factor q , we made an inessential change to possible range of our experience estimates $e_{ij}(n)$, allowing only values $1 \leq e_{ij}(n) \leq M$, with $M \gg 1$. We set $1 \leq e_{ij}(n) \leq 100$. Then, by our Averaging axiom, also the values $T(\vec{s}_{ij}(n))$ are in the same range. Consider now the mapping F given by

$$F : (\rho_i : i \leq N) \mapsto \left(\sum_{i \in \Delta_j(n)} \frac{f(w_{ij}(n), \rho_i) T(\vec{s}_{ij}(n))}{\sum_{k \in \Delta_j(n)} f(w_{kj}(n), \rho_k)} : j \leq N \right).$$

Since for every j the weights in the weighted j^{th} sum add up to one and since the scaling function f is continuous in both variables, it is easy to see that F is a continuous mapping of the N -dimensional cube $[1, M]^N$ into itself. Thus, since F is a continuous map from a convex and compact subset of R^N into itself, by the Brouwer Fixed point theorem F must have a fixed point, see e.g., [3], which we take as the values of $(\rho_j(n) : j \leq N)$. The fixed point can be obtained by standard iterative procedures. Our experimental results showed that the method is both fast and numerically robust.

Finally, we can evaluate the community evidence $(W_j(n) : j \leq N)$ using

$$W_j(n) = \sum_{k \in \Delta_j(n)} f(w_{kj}(n), \rho_k); j \leq N. \quad (5)$$

Thus, at each moment n and for each $j \leq N$, $\rho_j(n)$ is a representation and an estimate for our informal notion of community reputation enjoyed by agent A_j , and $W_j(n)$ is a community based measure of reliability of such reputation estimate.

Conclusion. Tennenholtz's [7] idea on the reputation of agents can be paraphrased as follows. Say an agent A_j is *supported* by a set S_j of other agents if the agents in S_j provide high trust rankings for A_j . Since his discussion is about a static domain there is no notion of time progression. Moreover, he argued that support by agents which themselves have high reputations should result in higher reputation for the supported agent than support by low reputation agents. We feel that we have provided an adequate and rigorous formalization of this idea, also extending Tennenholtz's static setup to one that involves time sequences of transactions.

One might ask a question how one should use reputation rank and the community evidence to make decision. For example, given to agent, one with reputation of 50 and community evidence 0.5 and another with reputation of 60 and community evidence 0.4, which should one choose? The answer depends on the particular features of the community and must be probabilistic in nature, in the sense that agents will act so that they maximize the *expected* utility. This problem, together with software engineering of such a system, e.g. clock reset, partition of market into overlap sub communities to avoid explosion in size of equation system, require further investigation.

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