

# Counterexample Guided Path Reduction for Static Program Analysis

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**Abstract.** In this work we introduce counterexample guided path reduction based on interval constraint solving for static program analysis. The aim of this technique is to reduce the number of false positives by reducing the number of feasible paths in the abstraction iteratively. Given a counterexample, a set of observers is computed which exclude infeasible paths in the next iteration. This approach combines ideas from counterexample guided abstraction refinement for software verification with static analysis techniques that employ interval constraint solving. The advantage is that the analysis becomes less conservative than static analysis, while it benefits from the fact that interval constraint solving deals naturally with loops. We demonstrate that the proposed approach is effective in reducing the number of false positives, and compare it to other tools for C/C++ program analysis.

## 1 Introduction

Static program analysis and software model checking are two automatic analysis techniques to ensure (limited) correctness of software or at least to find as many bugs in the software as possible. In contrast to software model checking, static program analysis typically works on a more abstract level, such as the control flow graph (CFG) without any data abstraction. As such a syntactic model of a program is a very coarse abstraction, and reported error traces can be spurious, i.e. they may correspond to no actual run in the concrete program [1]. The result of such infeasible paths are false positives in the program analysis.

This work presents counterexample guided path reduction to remove infeasible paths. To do so semantic information in the form of interval equations is added to a previously purely syntactic model. This is similar to abstract interpretation based interval analysis, e.g., used for buffer overflow detection. That approach transforms the entire program into a set of interval equations, and characterizes its behavior by the least fixpoint solution [2].

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Our approach, in contrast, constructs this kind of equation system only for a given path through the program. This is not only computationally cheaper, but it has the welcome side effect that the analysis becomes less conservative. The paths themselves are determined by potential counterexamples. A counterexample path is *spurious* if the least fixpoint of a corresponding equation system is empty. The subset of equations responsible for an empty solution is called a *conflict*. In a second step we refine our syntactic model by a finite *observer* which excludes all paths that generate the same conflict. These steps are applied iteratively, until either no new counterexample can be found, or until a counterexample is found that cannot be proven to be spurious.

This approach is obviously inspired by counterexample guided abstraction refinement (CEGAR), as used in [3, 4]. An important difference is the use of fixpoint computation for interval equations [2] in the place of SAT-solving. This technique deals directly with loops, without the need to discover additional loop-predicates [4, 5], or successive unrolling of the transition relation [6]. The proposed approach combines nicely with the static analysis approach put forward in [7] and implemented in the tool Goanna. It defines, in contrast to semantic software model checking, syntactic properties on a syntactic abstraction of the program.

The experimental results confirm that the proposed path reduction technique situates our tool in-between software model checking and static analysis tools. False positives are effectively reduced, while interval solving converges quickly, even in the presence of loops.

The next section introduces preliminaries of labeled transition systems, interval solving and model checking for static properties. Section 3 introduces Interval Automata, the model we use to capture the program semantics. Section 4 presents the details of our approach. Implementation details and a comparison with other tools are given in Section 5.

## 2 Preliminaries

### 2.1 Labeled Transition Systems

In this work we use *labeled transition systems* (LTS) to describe the semantics of our abstract programs. An LTS is defined by  $(S, S_0, A, R, F)$  where  $S$  is a set of states,  $S_0 \subseteq S$  is a sub-set of initial states,  $A$  is a set of actions and  $R \subseteq S \times A \times S$  is a transition relation where each transition is labeled with an action  $a \in A$ , and  $F \subseteq S$  is a set of final states. We call an LTS *deterministic* if for every state  $s \in S$  and action  $a \in A$  there is at most one successor state such that  $(s, a, s') \in R$ .

The finite sequence  $\rho = s_0 a_0 s_1 a_1 \dots a_{n-1} s_n$  is an *execution* of an LTS  $P = (S, S_0, A, R, F)$ , if  $s_0 \in S_0$  and  $(s_i, a_i, s_{i+1}) \in R$  for all  $i \geq 0$ . An execution is *accepting* if  $s_n \in F$ . We say  $w = a_0 \dots a_{n-1} \in A^*$  is a *word in P*, if there exist  $s_i, i \geq 0$ , such that  $s_0 a_0 s_1 a_1 \dots a_{n-1} s_n$  form an execution in  $P$ . The *language* of  $P$  is defined by the set of all words for which there exists an accepting execution. We denote this language as  $\mathcal{L}_P$ .

The *product* of two labeled transition systems  $P_1 = (S_1, S_{1_0}, A, R_1, F_1)$  and  $P_2 = (S_2, S_{2_0}, A, R_2, F_2)$ , denoted as  $P_\times = P_1 \times P_2$ , is defined as  $P_\times = (S_1 \times S_2, S_{1_0} \times S_{2_0}, A, R_\times, F_1 \times F_2)$  where  $((s_1, s_2), a, (s'_1, s'_2)) \in R_\times$  if and only if  $(s_1, a, s'_1) \in R_1$  and  $(s_2, a, s'_2) \in R_2$ . The language of  $P_\times$  is the *intersection* of the language defined by  $P_1$  and  $P_2$ .

## 2.2 Interval Equation Systems

We define an *interval lattice*  $\mathcal{I} = (I, \subseteq)$  by the set  $I = \{\emptyset\} \cup \{[z_1, z_2] \mid z_1 \in \mathbb{Z} \cup \{-\infty\}, z_2 \in \mathbb{Z} \cup \{\infty\}, z_1 \leq z_2\}$  with the partial order implied by the *contained in* relation " $\subseteq$ ", where a non-empty interval  $[a, b]$  is *contained in*  $[c, d]$ , if  $a \geq c$  and  $b \leq d$ . The empty element is the bottom element of this lattice, and  $[-\infty, +\infty]$  the top element. Moreover, we consider the following operators on intervals: addition (+), multiplication ( $\cdot$ ), union  $\sqcup$ , and intersection  $\sqcap$  with the usual semantics  $\llbracket \cdot \rrbracket$  defined on them.

For a given finite set of variables  $X = \{x_0, \dots, x_n\}$  over  $\mathcal{I}$  we define an *interval expression*  $\phi$  as follows:

$$\phi \doteq a \mid x \mid \phi \sqcup \phi \mid \phi \sqcap \phi \mid \phi + \phi \mid \phi \cdot \phi$$

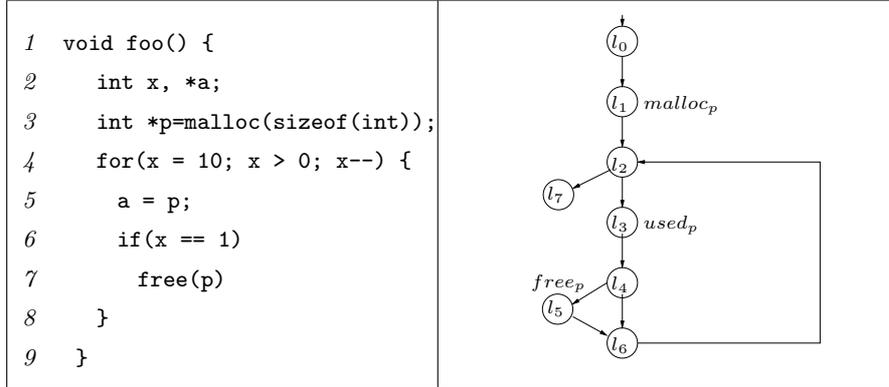
where  $x \in X$ , and  $a \in \mathcal{I}$ . The set of all expression over  $X$  is denoted as  $\mathcal{C}(X)$ .

For all operation we have that  $\llbracket \phi \circ \varphi \rrbracket$  is  $\llbracket \phi \rrbracket \circ \llbracket \varphi \rrbracket$ , where  $\circ$  can be any of  $\sqcup, \sqcap, +, \cdot$ . A *valuation* is a mapping  $v : X \rightarrow \mathcal{I}$  from an interval variable to an interval. Given an interval expression  $\phi \in \mathcal{C}(X)$ , and a valuation  $v$ , the  $\llbracket \phi \rrbracket_v$  denoted the expression  $\phi$  evaluated in  $v$ , i.e. it is defined to be the interval  $\llbracket \phi[v(x_0)/x_0, \dots, v(x_n)/x_n] \rrbracket$ , which is obtained by substituting each variable  $x_i$  with the corresponding interval  $v(x_i)$ .

An *interval equation system* is a mapping  $IE : X \rightarrow \mathcal{C}(X)$  from interval variables to interval expressions. We also denote this by  $x_i = \phi_i$  where  $i \in 1, \dots, n$ . The *solution* of such an interval equation system is a valuation satisfying all equations, i.e.,  $\llbracket x_i \rrbracket = \llbracket \phi_i \rrbracket_v$  for all  $i \in 1, \dots, n$ . As shown in [2] there always is a *precise least solution* which can be efficiently computed. By precise we mean precise with respect to the interval operators's semantics and without the use of additional widening techniques. Of course, from a program analysis point of view over-approximations are introduced, e.g., when joining two intervals  $[1, 2] \sqcup [4, 5]$  results in  $[1, 5]$ . This, however, is due to the domain we have chosen.

## 2.3 Static Analysis by Model Checking

This work is based on an automata based static analysis framework as described in [7], which is related to [8–10]. The basic idea of this approach is to map a C/C++ program to its CFG, and to label this CFG with occurrences of syntactic constructs of interest. The CFG together with the labels can easily be mapped to the input language of a model checker, in our case NuSMV, or directly translated into a Kripke structure for model checking.



**Fig. 1.** Example program and labeled CFG for use-after-free check.

A simple example of this approach is shown in Fig. 1. Consider the contrived program `foo` which is allocating some memory, copying it a number of times to `a`, and freeing the memory in the last loop iteration.

One example of a property to check is whether after freeing some resource, it still might be used. In our automata based approach we syntactically identify program locations that allocate, use, and free resource  $p$ . We automatically label the program's CFG with this information as shown on the right hand side of Fig. 1. This property can then be checked by the CTL property

$$AG (malloc_p \Rightarrow AG (free_p \Rightarrow \neg EF used_p)),$$

which means that whenever there is `free` after `malloc` for a resource  $p$ , there is no path such that  $p$  is used later on. Obviously, neglecting any further semantic information will lead to a false alarm in this example.

### 3 Interval Automata

This section introduces *interval automata* (IA) which abstract programs and capture their operational semantics on the domain of intervals. We define an IA as an extended state machine where the control structure is a finite state machine, extended by a mapping from interval variables to interval expressions. We will show later how to translate a C/C++ program to an IA.

**Definition 1 (Syntax).** An interval automaton is a tuple  $(L, l_0, X, E, update)$ , with

- a finite set of locations  $L$ ,
- an initial location  $l_0$ ,
- a set of interval variables  $X$ ,
- a finite set of edges  $E \subseteq L \times L$ , and
- an effect function  $update : E \rightarrow (X \times \mathcal{C}(X))$ .

The effect *update* assigns to each edge a pair of an interval variable and an interval expression. We will refer to the (left-hand side) variable part  $update|_X$  as *lhs*, and to the (right-hand side) expression part  $update|_{\mathcal{C}(X)}$  as *rhsexpr*. The set of all variables that appear in *rhsexpr* will be denoted by *rhsvars*. Note, that only one variable is updated on each edge. This restriction is made for the sake of simplicity, but does not restrict the expressivity of an IA. Fig. 2 shows an example of an IA.

**Definition 2 (Semantics).** *The semantics of an IA  $P = (L, \mathbf{l}_0, X, E, update)$  are defined by a labeled transition system  $LTS(P) = (S, S_0, A, R, F)$ , such that*

- the set of states  $S$  contains all states  $(l, v)$  with location  $l$ , and a interval valuation  $v$ .
- the set of initial states  $S_0$  contains all states  $s_0 = (\mathbf{l}_0, v_0)$ , with  $v_0 \equiv [-\infty, \infty]$ .
- the alphabet  $A$  is the set of edges  $E$ .
- the transition relation  $R \subseteq S \times A \times S$  contains a triple  $((l, v), (l', v'), (l', v'))$ , i.e. a transition from state  $(l, v)$  to  $(l', v')$  labeled  $(l, l')$ , if there exists a  $(l, l')$  in  $E$ , such that  $v' = v[lhs(e) \leftarrow \llbracket rhsexpr(e) \rrbracket_v]$  and  $\llbracket rhsexpr(e) \rrbracket_{v'} \neq \emptyset$ .
- the set of final states  $F = S$ , i.e. all states are final states

It might seem a bit awkward that the transitions in the LTS are labeled with the edges of the IA, but this will be used later to define the synchronous composition with an observer. Since each transition is labeled with its corresponding edge we obtain a deterministic system, i.e., for a given word there exists only one possible run. We identify a word  $((l_0, l_1), (l_1, l_2), \dots, (l_{m-1}, l_m))$  in the remainder by the sequence of locations  $(l_0, \dots, l_m)$ .

Given an IA  $P$ . Its language  $\mathcal{L}_P$  contains all sequences  $(l_0, \dots, l_n)$  which satisfy the following:

$$l_0 = \mathbf{l}_0 \tag{1}$$

$$\wedge \forall i = 0, \dots, n-1. (l_i, l_{i+1}) \in E \tag{2}$$

$$\wedge v_0 \equiv [-\infty, +\infty] \wedge \exists v_1, \dots, v_n. (\llbracket rhsexpr(l_i, l_{i+1}) \rrbracket_{v_i} \neq \emptyset \wedge v_{i+1} = v_i[lhs(l_i, l_{i+1}) \leftarrow \llbracket rhsexpr(l_i, l_{i+1}) \rrbracket_{v_i}]) \tag{3}$$

This mean that a word (1) starts in the initial location, (2) respects the edge relation  $E$ , and (3) that there exists a sequence of non-empty valuations that satisfies the updates associated with the edges. We use this characterization of words as satisfiability problem to generate systems of interval equations that have a non-empty solution only if a sequence  $(l_0, \dots, l_n)$  is a word. We will define for a given IA  $P$  and sequence  $w$  a *conflict* as an interval equation system with an empty least solution which proves that  $w$  cannot be a word of the IA  $P$ .

## 4 Path Reduction

The labeled CFG as defined in Section 2.3 is a coarse abstraction of the actual program. Like most static analysis techniques this approach suffers from false

positives. In the context of this paper we define a property as a regular language, and satisfaction of a property as language inclusion. The program itself will be defined by an Interval Automaton  $P$  and its behavior is defined by the language of the corresponding  $LTS(P)$ . Since interval automata are infinite state systems, we do not check the IA itself but an abstraction  $\hat{P}$ . This abstraction is initially an annotated CFG as depicted in Fig. 1.

A *positive* is a word in the abstraction  $\hat{P}$  that does not satisfy the property. A *false positive* is a positive that is not in the actual behavior of the program, i.e. it is not in the language of the  $LTS(P)$ . *Path reduction* is then defined as the iterative process that restricts the language of the abstraction, until either a true positive has been found, or until the reduced language satisfies the property.

#### 4.1 Path Reduction Loop

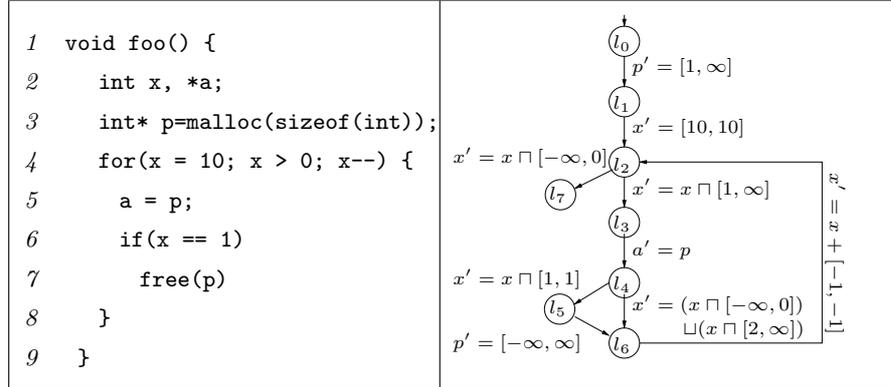
Given an IA  $P = (L, \mathbf{l}_0, E, X, update)$  we define its finite abstraction  $\hat{P}$  as follows:  $\hat{P} = (L, \mathbf{l}_0, E, E', L)$  is a labeled transition system with states  $L$ , initial state  $\mathbf{l}_0$ , alphabet  $E$ , transition relation  $E' = \{(l, (l, l'), l') \mid (l, l') \in E\}$ , and the entire set  $L$  as final states. The LTS  $\hat{P}$  is an abstraction of the  $LTS(P)$ , and it represents the finite control structure of  $P$ . The language of  $\hat{P}$  will be denoted by  $\mathcal{L}_{\hat{P}}$ . Each word of  $\hat{P}$  is by construction a word of  $LTS(P)$ . Let  $\mathcal{L}_\phi$  be the language of the specification.

We assume to have a procedure that checks if the language of LTS  $\mathcal{L}_{\hat{P}}$  is a subset of  $\mathcal{L}_\phi$ , and produces a counterexample if this is not the case (cf. Section 4.5). If it finds a word in  $\mathcal{L}_{\hat{P}}$  that is not in  $\mathcal{L}_\phi$ , we have to check whether this word is in  $\mathcal{L}_P$ , i.e. we have to check whether it satisfies equation (1) to (3). Every word  $w = (l_0, \dots, l_m)$  in  $\mathcal{L}_{\hat{P}}$  satisfies by construction (1) and (2). If a word  $w = (l_0, \dots, l_m)$  can be found such that there exists no solution for (3), it cannot be a word of  $\mathcal{L}_P$ . In this case we call the word *spurious*.

If we assume in addition to have a procedure to check whether a word is spurious (cf. Section 4.2), we can use these in an iterative loop to check if the infinite LTS of IA  $P$  satisfies the property, by checking a finite product of abstraction  $\hat{P}$  with observers instead as follows:

1. Let  $\hat{P}_0 := \hat{P}$ , and  $i = 0$ .
2. Check if  $w \in \mathcal{L}_{\hat{P}_i} \setminus \mathcal{L}_\phi$  exists. If such a  $w$  exists goto to step 3, otherwise exit with “property satisfied”.
3. Check if  $w \in \mathcal{L}_P$ . If  $w \notin \mathcal{L}_P$  build observer  $Obs_w$ , otherwise exit with “property not satisfied”. The observer satisfies the following
  - (a) it accepts  $w$ , and
  - (b) all accepted words  $w' \notin \mathcal{L}_P$ .
4. Let  $\hat{P}_{i+1} := \hat{P}_i \times Obs_w^C$ , with  $\cdot \hat{P}_i \times Obs_w^C$  is the synchronous composition of  $\hat{P}_i$  and the complement of  $Obs_w$ . Increment  $i$  and goto step 3.

The remainder of this section explains how to check if a word is in  $\mathcal{L}_P$ , how to build a suitable observer, and how to combine it in a framework which uses NuSMV to model check the finite abstraction  $\hat{P}_i$ .



**Fig. 2.** Abstraction of the program as IA. Analysis of the syntactic properties of the annotated CFG in Fig.1 is combined with an analysis of the IA to the right.

*Example.* The initial coarse abstraction as CFG in Fig. 1 loses the information that  $p$  cannot be used after it was freed. The shortest counterexample based on the CFG is to initialize  $x$  to 10 in line 4, enter the `for`-loop, take the `if`-branch, free  $p$  in line 7, decrement  $x$ , return to the beginning of the `for`-loop, and then to use  $p$  in line 5. This counterexample is obviously spurious. Firstly, because the `if`-branch with condition  $x == 1$  at line 7 is not reachable while  $x = 10$ . Secondly, because if the program could enter the `if`-loop, it would imply  $x == 1$ , and it would be impossible to reenter the loop, given decrement  $x--$  and loop condition  $x > 0$ .

## 4.2 Checking for Spurious Words

Every word  $w = (l_0, \dots, l_m)$  in  $\mathcal{L}_{\hat{P}}$  satisfies by construction (1) and (2). It remains to be checked if condition (3) can be satisfied. A straightforward approach is to execute the trace on  $LTS(P)$ . However this can only determine if that particular word is spurious. Our proposed approach builds an equation system instead, which allows us to find a set of conflicting interval equations, which can in turn be used to show that an entire class of words is spurious. Another straightforward approach to build such an equation system is to introduce for each variable and edge in  $w$  an interval equation, and to use an interval solver to check if a solution to this system of interval equations exists. A drawback of this approach is that it introduces  $m \times (n + 1)$  variables, and  $m \times n$  equations. In the following we present an approach to construct an equation system with at most one equation and one variable for each edge in  $w$ .

**Interval Equation System for a Sequence of Transitions.** We describe how to obtain an equation system for a word  $w \in \mathcal{L}_{\hat{P}}$ , such that it has a non-empty least solution only if  $w \in \mathcal{L}_P$ . This system is generated in three steps:

$w$	$var$	$expr_w(i)$	$IE_w$	Reduced conflicts	
$(l_0, l_1)$	$p_{l_1}$	$[1, \infty]$	$ \left. \begin{aligned} p_{l_1} &= [1, \infty] \\ x_{l_2} &= [10, 10] \sqcup \\ &\quad (x_{l_5} + [-1, -1]) \\ x_{l_3} &= (x_{l_2} \sqcap [1, \infty]) \\ a_{l_4} &= p_{l_1} \sqcup p_{l_6} \\ x_{l_5} &= x_{l_3} \sqcap [1, 1] \\ p_{l_6} &= [-\infty, \infty] \end{aligned} \right\} $	$ \left. \begin{aligned} x_{l_2} &= [10, 10] \\ x_{l_3} &= x_{l_2} \sqcap [1, \infty] \\ x_{l_5} &= x_{l_3} \sqcap [1, 1] \end{aligned} \right\} $	conflict 1
$(l_1, l_2)$	$x_{l_2}$	$[10, 10]$			
$(l_2, l_3)$	$x_{l_3}$	$x_{l_2} \sqcap [1, \infty]$			
$(l_3, l_4)$	$a_{l_4}$	$p_{l_1}$			
$(l_4, l_5)$	$x_{l_5}$	$x_{l_3} \sqcap [1, 1]$			
$(l_5, l_6)$	$p_{l_6}$	$[-\infty, \infty]$		$ \left. \begin{aligned} x_{l_5} &= [-\infty, \infty] \sqcap [1, 1] \\ x_{l_2} &= x_{l_5} + [-1, -1] \\ x_{l_3} &= x_{l_2} \sqcap [1, \infty] \end{aligned} \right\} $	conflict 2
$(l_6, l_2)$	$x_{l_2}$	$x_{l_5} + [-1, -1]$			
$(l_2, l_3)$	$x_{l_3}$	$x_{l_2} \sqcap [1, \infty]$			
$(l_3, l_4)$	$a_{l_4}$	$p_{l_6}$			

**Fig. 3.** Equations for counterexample  $w$  of the IA depicted in Fig. 2

*I. Tracking variables.* Let  $X_L$  be a set of fresh variables  $x_l$ . For each update of a variable, i.e.  $x = lhs(l_{i-1}, l_i)$  we introduce a fresh variable  $x_{l_i}$ . We use the notation  $x_{(i)}$  to refer to the last update of  $x$  before the  $i$ -th transition in  $w = (l_0, \dots, l_m)$ . If no such update exist  $x_{(i)}$  will be  $x_{l_0}$ .

*II. Generating equations.* For each edge in  $w$  we generate an interval expression over  $X_L$ . We define  $expr_w : \{0, \dots, m\} \rightarrow \mathcal{C}(X_L)$  as follows:

$$expr_w(i) \mapsto rhsexpr(l_{i-1}, l_i)[x_{(i-1)}/x]_{x \in rhsvars(l_{i-1}, l_i)} \quad (4)$$

An expression  $expr_w(i)$  is the right-hand side expression of the update on  $(l_{i-1}, l_i)$ , where all occurring variables are substituted by variables in  $X_L$ .

*III. Generating equation system.* Locations may occur more than once in a word, and variables maybe updated by multiple edges. Let  $writes_w \subseteq X_L$  the set  $\{x_l \exists i \text{ s.t. } x = lhs(l_{i-1}, l_i)\}$ , and  $\Omega_w$  be a mapping each  $x_l \in writes_w$  the set  $\{i | x = lhs(l_{i-1}, l_i) \wedge l_i = l\}$ . The system  $IE_w : X_L \rightarrow \mathcal{C}(X_L)$  is defined as follows:

$$x_l \mapsto \begin{cases} \bigsqcup_{i \in \Omega_w(x_l)} expr_w(i) & \text{if } x_l \in writes_w \\ [-\infty, \infty] & \text{otherwise} \end{cases} \quad (5)$$

System  $IE_w$  assigns each variable  $x_l \in writes_w$  to a union of expressions; one expression for each element in  $\Omega_w(x, l)$ .

*Example.* Fig. 3 depicts for word  $w = (l_0, l_1, l_2, l_3, l_4, l_5, l_6, l_2, l_3, l_4)$  how to generate  $IE_w$ . The first column gives the transitions in  $w$ . The second column gives the variable in  $writes_w$ . The variable  $p_{l_1}$ , for example, refers to the update of  $p$  on the first transition  $(l_0, l_1)$  of the IA in Fig. 2. We have that  $x_{(5)} = x_{l_3}$ , since the last update of  $x$  before  $(l_4, l_5)$  was on edge  $(l_2, l_3)$ .

The third column gives the equations  $expr_w(i)$ . For example, the right-hand side  $rhsexpr(l_4, l_5)$  is  $x' = x \sqcap [1, 1]$ . Since  $x_{(4)} = x_{l_3}$ , we get that  $expr_w(5)$  is  $x_{l_3} \sqcap [1, 1]$ . The fourth column shows the equation system  $IE_w$  derived from the equations. We have, for example, that  $x$  is updated on  $(l_1, l_2)$ , the second edge

in  $w$ , and  $(l_6, l_2)$ , the 8th edge. Hence,  $\Omega_w(x_{l_2}) = \{2, 8\}$ . Equation  $IE_w(x_{l_2})$  is then defined as the union of  $expr_w(2)$ , which is  $[10, 10]$ , and  $expr_w(8)$ , which is  $x_{l_5} + [-1, -1]$ . The least solution of the equation system  $IE_w$  is  $p_{l_1} = [1, \infty]$ ,  $x_{l_2} = [10, 10]$ ,  $x_{l_3} = [10, 10]$ ,  $a_{l_4} = [-\infty, \infty]$ ,  $x_{l_5} = \emptyset$ , and  $p_{l_6} = [-\infty, \infty]$ . Since  $x_{l_5} = \emptyset$ , there exists no solution, and  $w$  is spurious.  $\square$

**Lemma 1.** *Given a word  $w \in \mathcal{L}_{\hat{P}}$ . Then there exist a sequence of non-empty valuations  $v_1, \dots, v_m$  such that (3) holds for  $w$  only if  $IE_w$  has a non-empty least solution.*

*Proof.* Given solution to (3) we can construct non-empty solution of  $IE_w$ , which must be included in the least solution of  $IE_w$ .  $\square$

An advantage of the interval solving is that it can reason over loops, rather than linear traces. This is captured by the following. A third or fourth repetition does not introduce new variable in  $writes_w$ , and neither new expressions. This means that the if equation system for a word, which is a concatenation  $\alpha\beta\beta\gamma$ , has no non-empty solution, then the concatenation  $\alpha\beta\beta\beta\gamma$  will also no non-empty solution. See Lemma 2 in the appendix for a formal treatment of this observation.

### 4.3 Conflict Discovery

The previous subsection described how to check if a given word  $w \in \mathcal{L}_{\hat{P}}$  is spurious. Interval solving, however, leads to an over-approximation, mostly due to the  $\sqcup$ -operation. This subsection describes how to reduce the numbers of (non-trivial) equations in a conflict and at the same time the over-approximation error, by restricting conflicts to fragments and cone-of influence.

For CEGAR approaches for infinite-state systems it has been observed that it is sufficient and often more efficient to find a spurious fragments of a counterexample, rather than a spurious counterexample [11, 12]. The effect is similar to small or minimal predicates in SAT-based approaches. The difference between a word and a fragment in our context is that a fragment may start in any location.

Given a word  $w = (l_0, \dots, l_m)$  a fragment  $w' = (l'_0, \dots, l'_{m'})$  is a subsequence of  $w$ . For fragment  $w'$  of a word  $w$  we can show the analog of Lemma 1. If the least solution of  $IE_{w'}$  is empty, i.e. if the fragment  $w'$  is spurious, then the word  $w$  is spurious. If there exists a sequence of non-empty valuations  $v_1, \dots, v_m$  for  $w$ , then they also define a non-empty subsequence of valuations for  $w'$ , and these have to be contained in the least solution of  $IE_{w'}$ .

A word of length  $m - 1$  has  $m^2/2$  fragments. Rather than checking all of these we can rule out most of these based on two observations. To be useful at least the last edge of a fragment has to result in an empty solution. An *update* in (3) can only result in an empty solution, if there exist an element in  $\mathcal{I}$  that is mapped to the empty set by *update*. We call such updates *restricting*. An example of such updates are intersections with constants such as  $x' = x \sqcap [1, \infty]$ . We can omit all  $(l_{i-1}, l_i)$  after the last restricting update in  $w'$ . Similarly,

we find that edges with updates that map valuation  $v \equiv [-\infty, \infty]$  to  $[-\infty, \infty]$ , can be omitted from the beginning of a fragment.

Given thus reduced fragment  $w'$ , we can reduce  $IE_{w'}$  further, by omitting all equations that are not in the cone-of-influence of the last restricting update. We refer to the resulting system of equations as  $\overline{IE}_{w'}$ . In the remainder we will refer to  $\overline{IE}_{w'}$  as the *reduced conflict*, which is uniquely determined by the fragment  $w'$ . The reduction to the cone-of-influence, guarantees that  $IE_{w'}$  has an empty least solution if  $\overline{IE}_{w'}$  has. The converse is not true. However, since we consider all possible fragments, it is guaranteed that if a set of removed equations result in an conflict, these will appear as reduced conflict of another fragment.

*Example.* There are two conflicts among the candidate fragments in Fig. 3. Conflict 1, for fragment  $(l_1, l_2, l_3, l_4, l_5)$ , has as least solution  $x_{l_2} = [10, 10]$ ,  $x_{l_3} = [10, 10]$ ,  $x_{l_5} = \emptyset$ . Conflict 2, for fragment  $(l_4, l_5, l_6, l_2, l_3)$ , has as least solution  $x_{l_5} = [1, 1]$ ,  $x_{l_2} = [0, 0]$ ,  $x_{l_3} = \emptyset$ . The equation was  $p_{l_6}$  was not included in the second conflict, as it is not in the cone-of-influence of  $x_{l_3}$ . Note, that variable  $x$  in equation  $\overline{IE}(x_{l_5})$  was substituted by  $[-\infty, \infty]$ . This expression was generated by the first edge  $(l_4, l_5)$  of the fragment  $(l_4, l_5, l_6, l_2, l_3)$ . The last update before this edge is outside of the scope of the fragment, and hence  $x$  was assumed to be  $[-\infty, \infty]$ , which is a conservative over-approximation.

#### 4.4 Conflict Observer

Given a reduced conflict  $\overline{IE}_w$  for a fragment  $w = (l_0, \dots, l_m)$ , we construct an observer such that if a word  $w' \in \mathcal{L}_{\hat{P}}$  is accepted, then  $w' \notin \mathcal{L}_P$ . The observer is a LTS over the same alphabet  $E$  as  $LTS(P)$  and  $\hat{P}$ .

**Definition 3.** *Given an IA  $P = (L, \mathbf{l}_0, E, X, \text{update})$  and reduced conflict  $\overline{IE}_w$  for a fragment  $w = (l_0, \dots, l_m)$ , define  $X_w$  as the set of all variables  $x \in X$  such that  $x = \text{lhs}(l_{i-1}, l_i)$ , for some edge  $(l_{i-1}, l_i)$  in  $w$ . The observer  $Obs_w$  is a LTS with*

- set  $S_{Obs}$  of states (current, eqn, conflict) with valuation  $\text{current} : X_w \rightarrow L$ , valuation  $\text{eqn} : \text{writes}_w \rightarrow \{\text{unsat}, \text{sat}\}$ , and location  $\text{conflict} \in \{\text{all}, \text{some}, \text{none}\}$ .
- initial state  $(\text{write}_0, \text{eqn}_0, \text{loc}_0)$  with  $\text{current}_0 \equiv \mathbf{l}_0$ ,  $\text{eqn}_0 \equiv \text{unsat}$ , and  $\text{conflict}_0 = \text{none}$ .
- alphabet  $E$
- transition relation  $S_{Obs} \times E \times S_{Obs}$ .
- a set final states  $F$ . A state is final if  $\text{conflict} = \text{all}$ .

Due to the limited space we give an informal definition of the transition relation. The detailed formal definition can be found on [?].

- Variable *conflict* changes its state based on the next state of *eqn*. We have that  $\text{conflict}'(x_l)$  is *all*, *some*, *none*, if  $\text{eqn}'(x_l) = \text{sat}$  for all, some or no  $x_l \in \text{writes}_w$ , respectively. Once *conflict* is in *all* it remains there forever.

- Variable  $eqn$  represents  $\overline{IE}_w(x_l)$  for  $x_l \in writes_w$ . The successor  $eqn'(x_l)$  will change if  $x = lhs(\lambda, \lambda')$  and  $\lambda' = l$ . If substituting variables  $x$  by  $x_{current(x)}$  in  $rhserpr(\lambda, \lambda')$ , results in an expression that does not appear in  $\overline{IE}_w(x_l)$ , for any  $x_l$ , then  $eqn'(x_l) = unsat$  for all  $x_l$ . Otherwise, the successor  $eqn'(x_l)$  is  $sat$  for matching variables  $x_l$ , and remains unchanged for all others. This is a simple syntactic match, achieved by comparing the indices and variables that appear in  $\overline{IE}_w(x_l)$  with the values of  $current(x)$ .
- Variable  $current$  is used to record for each variable the target location of the last update. If  $conflict'$  is  $none$ , then  $current'(x) = l_0$ . Otherwise, if  $(\lambda, \lambda')$  is in  $w$ , and  $x = lhs(\lambda, \lambda')$ , then  $current'(x) = \lambda'$ . Otherwise, it remains unchanged.

The interaction between  $current$ ,  $eqn$ , and  $conflict$  is somewhat subtle. The idea is that the observer is initially in  $conflict = none$ . If an edge is observed such which generates an expression  $expr(i)$  that appears in  $\overline{IE}_w(x_l)$ , with  $i \in \Omega_w(x_l)$  (see Eq. (5)), then  $conflict' = some$ , and the corresponding  $eqn(x_l) = sat$ . It can be deduced  $\overline{IE}_w(x_l)$  is satisfied, unless another expression is encountered that might enlarge the fix-point. This is the case when an expression for  $x_l$  will be generated, that does not appear in  $\overline{IE}_w(x_l)$ . It is conservatively assumed that this expression increases the fixpoint solution.

If  $conflict = all$  it can be deduced that the observed edges produce an equation system  $\overline{IE}_{w'}$  that has a non-empty solution only if  $\overline{IE}_w$  has a non-empty solution. And from the assumption we know that  $\overline{IE}_w$  has an empty-solution, and thus also  $\overline{IE}_{w'}$ . Which implies that the currently observed run is infeasible and cannot satisfy Eq. 3.

Given a reduced conflict  $\overline{IE}_w$  for fragment  $w$  we have two properties.

- If a word  $w' \in \mathcal{L}_{\hat{P}}$  is accepted by  $Obs_w$ , then  $w \notin \mathcal{L}_P$ .
- Given a fragment  $w'$  such that  $\overline{IE}_{w'} = \overline{IE}_w$ , and a word  $w''$  such that  $w'$  is a subsequence, then  $w''$  is accepted by  $Obs_w$ .

The first property states that the language of  $\hat{P}' = \hat{P} \times Obs_w^C$  contains  $\mathcal{L}_P$ . The second ensures that each observer is only constructed once. This is needed to ensure termination. Each observer is uniquely determined by the finite set of expressions that appear in it, and since  $X_L$  and  $E$  are finite, there exists only a finite set of possible expressions that may appear in (4). Consequently, there can only exist a finite set of conflicts as defined by (5).

*Example.* The observer for the first conflict in Fig. 3 accepts a word if a fragment generates conflict  $x_{l_2} \mapsto [10, 10]$ ,  $x_{l_3} \mapsto x_{l_2} \sqcap [1, \infty]$ ,  $x_{l_5} \mapsto x_{l_3} \sqcap [1, 1]$ . This is the case if it observes edge  $(l_1, l_2)$ , edge  $(l_2, l_3)$  with a last write to  $x$  at  $l_2$ , and edge  $(l_4, l_5)$  with a last write to  $x$  at  $l_3$ . All other edges are irrelevant, as long as they do not write to  $x_2$ ,  $x_3$  or  $x_5$ , and change the solution. This would be, for example, the case for  $(l_2, l_3)$  if  $current(x) \neq l_2$ . This would create an expression different from  $x_{l_2} \sqcap [1, \infty]$ , and thus potentially enlarge the solution set. The NuSMV model of this observer can be found in the appendix.

The observer for the other conflicts is constructed similarly. The complement of these observers are obtained by labeling all states in  $S \setminus F$  as final. The product of the complemented observers with the annotated CFG in Fig.1 removes all potential counterexamples. The observer for the first conflict prunes all runs that enter the `for`-loop once, and then immediately enter the `if`-branch. The observer for the second conflict prunes all words that enter the `if`-branch and return into the loop.

#### 4.5 Path Reduction with NuSMV

The previous subsections assumed a checker for language inclusion for LTS. In practice we use however the CTL model checker NuSMV. The product of  $\hat{P}$  with the complement of the observers is by construction proper a abstractions of  $LTS(P)$ . The results language inclusion would therefore extend also to invariant checking, path inclusion, LTL and ACTL model checking. For pragmatic reasons we do not restrict ourselves to any of these, but use full CTL and LTL<sup>1</sup>. Whenever NuSMV produces a counterexample path, we use interval solving as described before to determine if this path is spurious.

Note, that path reduction can also be used to check witnesses, for example for reachability properties. In this case path reduction will check if a property which is true on the level of abstraction is indeed satisfied by the program.

The abstraction  $\hat{P}$  and the observers are composed synchronously in NuSMV. The observer synchronizes on the current and next location of  $\hat{P}$ . The property is defined as a CTL property of  $\hat{P}$ . The acceptance condition of the complements of observers is modeled as LTL fairness condition  $G\neg(\text{conflict} = \text{all})$ .

## 5 Implementation and Experiments

### 5.1 C to Interval Equations

This section describes how to abstract a C/C++ program to a set of interval equations, and covers briefly expression statements, condition statements as well as the control structures.

*Expressions statements* involving simple operations such as addition and multiplication are directly mapped to interval equations. E.g., an expression statement `x=(x+y)*5` is represented as  $x_{i+1} = (x_i + y_i) * [5, 5]$ . Subtraction such as `x = x - y` can be easily expressed as  $x_{i+1} = x_i + ([-1, -1] * y_i)$ .

*Condition statements* occur in constructs such as if-then-else, for-loops, while-loops etc. For every condition such as `x<5` we introduce two equations, one for the true-case and one for the false-case. Condition `x<5` has two possible outcomes  $x_{tt} = x \sqcap [-\infty, 4]$  and  $x_{ff} = x \sqcap [5, \infty]$ . More complex conditions involving more than one variable can also be approximated and we refer the interested reader to [13].

<sup>1</sup> NuSMV 2.x supports LTL as well

	no violation					violation				
	loop $x = 10$	loop $x = 100$	clutter w/o loop	clutter with loop	inferred loop inv	loop $x = 10$	loop $x = 100$	clutter w/o loop	clutter with loop	inferred loop inv
Splint	+	+	-	-	-	-	-	+	+	+
UNO	(-)	(-)	(-)	(-)	(-)	(+)	(+)	+	+	+
com. tool	+	+	+	+	+	+	+	+	+	+
Goanna	+	+	+	+	+	+	+	+	+	+
Blast	+	+	+	+	+	+	+	+	+	+
Satabs	+	+	+	+	+	+	+	+	+	+
CBMC	+	+	+	+	+	+	+	+	+	+

	no violation					violation				
	loop $x = 10$	loop $x = 100$	clutter w/o loop	clutter with loop	inferred loop inv	loop $x = 10$	loop $x = 100$	clutter w/o loop	clutter with loop	inferred loop inv
Goanna	2	2	3	2	1	2	2	4	3	2
Blast	3	3	4	12	12	11	101	5	12	10
Satabs	4	4	1	1	21	10	100	3	1	19
CBMC	10	100	1	10	10	11	101	1	10	11

**Table 1.** The table on the left-hand side shows for each tool if it found a true positive/negative “+” or a false positive/negative “-”. Entries “(-)” and “(+)” refer to warnings that there *may* be an error. The table on the right-hand-side compares the number of iterations of the three software model checkers with Goanna.

*Joins* are introduced where control from two different branches can merge. For instance, let  $x_i$  be the last *lhs*-variable in the if-branch of an if-then-else, and let  $x_j$  be the last *lhs*-variables in the else-branch. The values after the if-then-else is then the union of both possible values, i.e.,  $x_k = x_i \sqcup x_j$ .

For all other operations which we cannot accurately cover we simply over-approximate their possible effect. Function calls, for example, are handled as conservatively as possible, i.e.,  $\mathbf{x}=\mathbf{foo}()$  is abstracted as  $x_i = [-\infty, +\infty]$ . The same holds for most of pointer arithmetic, floating point operations and division.

It should be noted that infeasible paths mostly depend on combinations of conditions which cannot be satisfied. Typically, condition expressions and the operations having an effect on them are rather simple. Therefore, it is a sufficient first approach to over-approximate most but the aforementioned constructs.

## 5.2 Comparison

To evaluate our path reduction approach we added it to our static checker Goanna, and compared its results with three other static analysis tools, and three software model checkers. One of the static analyzers is a commercial tool. We applied these tools to five programs.<sup>2</sup>

- The first program is the one discussed throughout the paper. It is similar to the example discussed in [5].
- The second program is identical, except that the loop is initialized to  $\mathbf{x}=100$ .
- The third program tests how different tools deal with unrelated clutter. The correctness depends on two if-conditions: the first sets a flag, and the second assigns a value to a variable only if the first is set. In between the two if-blocks is some unrelated code, in this case a simple if-then-else.

<sup>2</sup> These can be found on <http://www.cse.unsw.edu.au/~ansgar/sas/>

- The fourth program is similar to the third, except that a for-loop counting up to 10 was inserted in-between the two relevant if-blocks .
- The last one is similar to the first program. This program uses however two counter-variables  $x$  and  $y$ . The correctness of the program depends on the loop-invariant  $x = y$ . It is similar to the examples presented in [14].

For each of the programs we constructed one instance with a bug, and one without. For the first program, e.g. the loop condition was changed to  $x >= 0$ . Since not all tools check for the same, we introduced different bugs for the different tools, which however all depended on the same paths.

The results in Table 1 show the static analysis tools often fail to produce correct warnings. Splint, for example, produces for the instances with and without a bug the same warnings, which is only correct in half of the cases. The software model checking tools in contrast always give the correct result. Our proposed path reduction produces one false positive for the last program. The least solution of the interval equation shows that both variables take values in the same interval, but it cannot infer the stronger invariant  $x = y$ .

The experiments were performed on a DELL PowerEdge SC1425 server, with an Intel Xeon processor running at 3.4 GHz, 2 MiB L2 cache and 1.5 GiB DDR-2 400 MHz ECC memory. For test programs as small as these the run-time reflect mostly the time for overhead and setup, and the following run-times should be considered with care. The following table gives the maximal, minimal, mean and median run-times in seconds

	max	min	mean	median
Splint	0.012	0.011	0.012	0.012
UNO	0.032	0.025	0.028	0.025
com. tool	0.003	0.002	0.003	0.003
Goanna	0.272	0.143	0.217	0.226
Blast	58.770	0.126	6.371	0.529
Satabs	5374.037	0.076	539.694	0.336
CBMC	0.221	0.079	0.137	0.121

The static analysis tools are overall fast, and have run times that are almost independent of the example program. For the three model checkers the program with the loop which is initialized to  $x=100$  that did contain a bug. Satabs, in particular showed exponential slow down. The default for Satabs to abort after 50 iterations, which takes about 460 seconds. To run the example we doubled the maximal number of iterations results, which results in a more than 10 times increase in run time. Memory was not a problem, as Satabs never exceeded 5% of the available memory. Blast showed a similar slowdown, but less spectacular. CBMC, in contrast, slowed down about linearly. Even if the loop was initialized to  $x = 1000$ , CBMC needed just about 2.4 sec for 1000 iterations. The run time for those programs was independent of the number of loops for Goanna, as expected. Goanna benefits from the fact that interval solving deals with loops efficiently.

## 6 Conclusions

In this work we presented an approach to enhance static program analysis with counterexample guided path reduction to eliminate false positives. While by default we investigate programs on a purely syntactic level, once we find a potential counterexample, it is mapped to an interval equation system. In case that the least solution of this system is empty, we know that the counterexample is spurious and identify a subset of equations which caused the conflict. We create an observer for the conflict, augment our syntactic model with this observer, re-run the analysis with the new model and keep repeating this iterative process until no more counterexamples occur or none can be ruled out anymore.

One of the advantages of our approach is that we do not require to unroll loops or to detect loop predicates as done in some CEGAR approaches. In fact, path-based interval solving is insensitive to loop bounds, and handles loops just like any other construct. However, path-based interval solving adds precision to standard abstract interpretation interval analysis. Moreover, we only use one data abstraction, namely a simple interval semantics for C/C++ programs.

We implemented our approach in our static analysis tool Goanna and compared it for a set of examples to existing software model checkers and static program analyzers. This demonstrated that Goanna is typically more precise than standard program analyzers and almost as precise as static model checkers. Of course, the design and strength of software model checkers is to check for a much richer class of properties.

Future work is to evaluate our approach further on real life software to identify a typical false alarm reduction ratio. Given that static analysis typically turns up a few bugs per 1000 lines of code, this will require some extensive testing. Moreover, we like to explore if slightly richer domains can be used to get additional precision without a signification increase in computation time and, most importantly, if this makes any significant difference for real life software.

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## A Appendix

**Lemma 2.** *Given a word  $w \in \mathcal{L}_{\hat{P}}$ , with  $w$  being a concatenation  $\alpha\beta\beta\gamma$  of sequences  $\alpha = (l_0^\alpha, \dots, l_{m_\alpha}^\alpha)$ ,  $\beta = (l_1^\beta, \dots, l_{m_\beta}^\beta)$ , and  $\gamma = (l_1^\gamma, \dots, l_{m_\gamma}^\gamma)$ . Let  $w' = \alpha\beta\beta\beta\gamma$ . Then  $w' \in \mathcal{L}_{\hat{P}}$ ,  $writes_w = writes_{w'}$  and the least solution of  $IE_w$  is also the least solution of  $IE_{w'}$ .*

*Proof:* (i) Given  $w \in \mathcal{L}_{\hat{P}}$  we have for  $i = 0, \dots, m_\alpha + 2m_\beta + m_\gamma$  that  $(l_i, l_i) \in E$ . This implies that also all edges in  $w'$  are in  $E$ , and hence,  $w' \in \mathcal{L}_{\hat{P}}$ .

(ii) Observe, that the second and third repetition of  $\beta$  in  $w'$  add no new tuples  $(x, l)$  to  $writes_w$ , hence  $writes_w = writes_{w'}$ .

(iii) Let  $x \in rhsvars(l_{i-1}, l_i)$  and update in the second iteration of  $\beta$ , i.e.  $i \in m_\alpha + m_\beta + 1, \dots, m_\alpha + 2m_\beta$ . Then  $lwrt_w(x, i-1)$  either refers to locations in  $l_i$  with  $i \in 0, \dots, m_\alpha$  or  $i \in m_\alpha + 1, \dots, 2m_\alpha$ . In either case we can show that  $lwrt_w(x, i-1 + m_\beta) = lwrt_w(x, i-1)$ . Henceforth, the third iteration of the loop will lead to the same expressions  $rhsexpr(l_{i-1}, l_i)[x_{(lwrt_w(x, i-1))}/x]$ . Taking the union of more of the same expressions will not change the least solution.  $\square$

```

MODULE conflict1(pc)
VAR
  current_x: {10,11,12,13,14,15,16};
  eqn_x_12, eqn_x_13, eqn_x_15: boolean;
  conflict:{some, none,all};

ASSIGN
  init(current_x):=10;
  init(eqn_x_12):=0;
  init(eqn_x_13):=0;
  init(eqn_x_15):=0;
  init(loc):=none;

  next(conflict):=
    case
      loc=all:all;
      next(eqn_x_12) & next(eqn_x_13) & next(eqn_x_15): all;
      next(eqn_x_12) | next(eqn_x_13) | next(eqn_x_15): some;
      1:none;
    esac;

  next(eqn_x_12) :=
    case
      ((pc=16 & next(pc) = 12) | (pc = 12 & next(pc) = 13 & current_x != 12)
      | (pc = 14 & next(pc) = 15 & current_x != 13)): 0;
      pc = 11 & next(pc) = 12: 1;
      1: eqn_x_12;
    esac;

  next(eqn_x_13) :=
    case
      ((pc=16 & next(pc) = 12) | (pc = 12 & next(pc) = 13 & current_x != 12)
      | (pc = 14 & next(pc) = 15 & current_x != 13)): 0;
      pc = 12 & next(pc) = 13 & current_x=12:1;
      1: eqn_x_13;
    esac;

  next(eqn_x_15) :=
    case
      ((pc=16 & next(pc) = 12) | (pc = 12 & next(pc) = 13 & current_x != 12)
      | (pc = 14 & next(pc) = 15 & current_x != 13)): 0;
      pc = 14 & next(pc) = 15 & current_x=13:1;
      1: eqn_x_15;
    esac;

  next(current_x):=
    case
      next(conflict)=none:10;
      pc = 11 & next(pc) = 12:12;
      pc = 12 & next(pc) = 13:13;
      pc = 14 & next(pc) = 15:15;
      pc = 14 & next(pc) = 16:14;
      pc = 16 & next(pc) = 12:12;
      1:current_x;
    esac;

FAIRNESS ! (conflict=all)

```

**Table 2.** NuSMV model of the observer for conflict 1. Input pc is the location of abstraction  $\hat{P}$ .