Abstract—In this paper, we consider the problem of using wireless sensor networks (WSNs) to measure the temporal-spatial profile of some physical phenomena. We base our work on two observations. Firstly, most physical phenomena are compressible in some transform domain basis. Secondly, most WSNs have some form of heterogeneity. Given these two observations, we propose a non-uniform compressive sensing method to improve the performance of WSNs by exploiting both compressibility and heterogeneity. We apply our proposed method to a real WSN data set. We find that our method can provide a more accurate temporal-spatial profile for a given energy budget compared with other sampling methods.

I. INTRODUCTION

In this paper, we consider the problem of using wireless sensor networks (WSNs) to measure the temporal-spatial profile of some physical phenomena in an energy efficient manner. Much work [7], [5], [17] has been done in improving the efficiency of WSNs in the past decade. The key distinction of this paper is that we exploit compressibility and heterogeneity to derive a non-uniform compressive sensing method to improve the performance of WSNs. More specifically, the non-uniform compressive sensing method that we propose can give a more accurate temporal-spatial profile for a given energy budget compared with other methods.

Our work is based on two hypotheses. Firstly, we assume that most physical phenomena are compressible in some transform domain basis. This is also the assumption behind the recently proposed theory of compressive sensing (CS), which is an efficient signal reconstruction method that can recover a signal from a small number of samples [3], [2]. This is also the assumption behind a number of recent work [1], [4] on using compressive sensing to improve the operations of WSNs. Secondly, we assume that each WSN has some form of heterogeneity. For example, in a multi-hop WSN, different nodes require different amount of energy to forward packets to the sink due to their relative position to the sink. Nodes that are far away from the sink will only need to relay few packets for other nodes while nodes close to the sink will need to relay more packets for other nodes [9]. It is also possible that nodes choose transmission power adaptively based on the local link quality observations [10], resulting in different nodes requiring different amount of energy to forward packets to the sink.

Given these two hypotheses, we propose a non-uniform compressive sensing (NCS) method to improve the performance of WSNs by exploiting both compressibility and heterogeneity, and evaluate the proposed NCS extensively with a real WSN application dataset, which features resource consumption heterogeneity. Furthermore, we present a distributed implementation of NCS framework that introduces very little communication overheads, and show that, compared to previously proposed approach based on and sparse approximation [14], NCS achieves similar signal approximation accuracy but with significantly less energy consumption.

The rest of this paper is organized as follows. Section II discusses assumptions (compressible signals and resource heterogeneity) and describes the proposed NCS architecture. In Section III, we discuss the basic set-up of the problem and the background knowledge of CS, which is followed by the introduction of the notion of NCS in Section IV. We evaluate and study proposed NCS framework by a dataset from a WSN deployment in Section V. We discuss prior work in Section VI. Finally, Section VII concludes the paper.

II. ASSUMPTIONS AND NCS ARCHITECTURE

A. Assumptions

WSNs are deployed to obtain an accurate temporal-spatial profile of some physical phenomena, e.g., temperature, humidity, wind speed, and/or wind direction [12], [18]. In this paper, we make the following two assumptions on the behavior of WSNs for NCS architecture.

Assumption 1: The signals (or physical phenomena) monitored by WSNs are compressible in some transform domain basis. Common examples of transform domains include DCT, wavelets or Haar wavelet.

Assumption 2: There is heterogeneity, e.g., the energy supply (harvest) rate and/or energy consumption rate, in the WSNs, and we can exploit the heterogeneity to increase the performance (e.g., increase network duty cycles and lifetime) of WSNs.
For example, energy-rich nodes can be sampled with higher rate compared to energy-constrained nodes, which may extend the battery life of energy-constrained nodes and, thereby the lifetime of the WSN can be extended.

We believe that these two assumptions are not restrictive at all and can be satisfied by most WSNs. We will introduce an application in Section V, which shows that NCS exploits the energy consumption heterogeneity of nodes to reduce network energy consumption.

B. NCS Architecture

Fig. 1 shows the architecture of NCS. The NCS middleware takes the application/user policy (signal accuracy requirement) as external input. Based on these inputs, the sample scheduler decides the sampling parameters, such as the number of samples that needs to be collected from a WSN and some global parameters such as network energy supply or consumption rates, and informs individual nodes. At a sampling instance, a node will decide whether to take a sample or not based on the output of a local random number generator with a non-uniform probability distribution, which takes the sampling parameters and some local information such as node energy supply or consumption rate as input. If a sample is taken by a node, the sample is forwarded to the base station. These samples are used to recover the monitored physical phenomena by the reconstruction module. When there is a significant difference between the number of collected samples and the number of requested samples, the reconstruction module will inform the sample scheduler which will in turn notify individual nodes by changing sample parameters.

An important argument that we will make in the following section is that, due to the heterogeneity of the WSNs, the probability distribution for making sampling decision should be non-uniform. Furthermore, we will demonstrate that non-uniform sampling has a better performance compared with uniform sampling.

III. BASIC SET-UP AND BACKGROUND ON COMPRESSIVE SENSING

We point out in the previous section that heterogeneity in resource supply or resource consumption is a common phenomenon in WSNs. Given this heterogeneity in resource supply or consumption in WSNs, we propose that non-uniform sampling can be a tool that we can exploit to improve the performance in such WSNs. In particular, we want to show that, by using non-uniform sampling, we can, for a given energy budget, improve the accuracy of the temporal-spatial profile obtained from WSNs.

In order to realise this goal, we need to develop a model to understand how non-uniform sampling affects the accuracy of the temporal-spatial profile obtained. If we have such a model and if we also know the energy consumption of a particular non-uniform sampling pattern, then we will be able to determine good sampling patterns that give accurate temporal-spatial profile for a given energy consumption. Therefore, in Section IV, we will develop a model to show how non-uniform sampling affects the accuracy of the temporal-spatial profile. The model that we will develop uses compressive sensing as the building block. The aim of this section is two-fold. Firstly, we present a basic set-up of the problem and review some results in compressive sensing which is necessary for the development in Section IV.

We begin with setting up the problem. Consider a wireless sensor network with $N$ nodes where each node measures a number of physical phenomena, e.g. temperature, humidity, wind speed, wind direction. We will consider one physical phenomenon at a time. Let $x_{it}$ denote the value of a particular physical phenomenon at sensor $i$ (where $i = 1, ..., N$) and time $t$ (where $t = 1, ..., T$). The complete temporal-spatial profile of a physical phenomenon consists of $n = NT$ values of $x_{it}$ with $i = 1, ..., N$ and $t = 1, ..., T$. It is obviously good to have the complete temporal-spatial profile since this provides the maximum amount of information. However, this means that all sensor nodes will need to sample at all time and this can result in high sampling or transmission energy consumption. In order to lower the energy consumption, we do not require all the sensors to sample the physical phenomenon at all time. If the value of a physical phenomenon $x_{j\tau}$ is not measured (or sampled) by sensor $j$ at time $\tau$, we will predict the value of $x_{j\tau}$ from those sensor readings that are available.

In the following discussion, we will collect all the values of $x_{it}$ into a $n \times 1$ vector $x$ where each element of $x$ corresponds to the value of the physical phenomenon at a particular sensor at a particular time. We will assume some of the elements of the vector $x$ are known (i.e. an element of $x$ is known if the corresponding sensor samples at the corresponding time) and the goal is to predict the unknown elements of $x$ from those that are known. A key idea behind the prediction method is to exploit the fact that most physical phenomena are compressible in some transform basis, e.g. Fourier, DCT, wavelets etc. A signal $x \in \mathbb{R}^n$ is said to be compressible in a transform basis $\Psi$, if the coefficients of the $x$ in the basis $\Phi$ decays according

![Fig. 1. System architecture.](image-url)
to the power law. In order to define this more precisely, we specify a transform basis $\Psi$ by a $n \times n$ matrix $\Psi$ whose columns are the basis vectors. In this case, the coefficients of $x$ in the basis $\Psi$ is given by the vector $g$ where $x = \Psi g$. Let us rearrange the elements in $g$ in decreasing order of magnitude, $|g_{(1)}| \geq |g_{(2)}| \geq \ldots \geq |g_{(n)}|$, then $x$ is compressible if the following condition holds:

$$|g_{(k)}| \leq Ck^{-p} \quad \forall k = 1, \ldots, n$$

(1)

for some $p \geq 1$ [3] and some constant $C$. We will use data collected from real wireless sensor networks to show that physical phenomena such as temperature and humidity are compressible in spatial domain.

We now explain how compressive sensing method, such as the one described in [2], can be used to estimate the unknown elements in $x$ from those that are known. We first introduce the concept of sampling matrix, denoted by $\Phi$. Let us assume that $m$ elements of $x$ are known and the indices of these $m$ elements in $x$ are $k_1, k_2, \ldots, k_m$. Let $\Omega$ be the set of the indices of the samples, i.e. $\Omega = \{k_1, k_2, \ldots, k_m\}$. Let also $I \in \mathbb{R}^{n \times n}$ denote the identity matrix. We define $I_{\Omega}$ be an $m$-by-$n$ matrix such that the $k$-th row of $I$ is also a row in $I_{\Omega}$ if $k \in \Omega$. With this definition, the vector $I_{\Omega}x \in \mathbb{R}^m$ contains the known elements of $x$.

The compressive sensing method in [2] says that one can estimate the unknown elements in $x$ given $I_{\Omega}x$ (i.e. the known elements in $x$) and the fact that $x$ is compressible in the transform domain $\Psi$ by solving the following linear programming problem:

$$\hat{x} = \Psi \hat{y} \text{ where } \hat{y} = \arg \min_{y \in \mathbb{R}^n} \|y\|_1 \text{ s.t. } I_{\Omega} \Psi y = I_{\Omega}x$$

(2)

For the case where the vector $x$ is sparse (i.e. most of the elements of the coefficients of $x$ in the transform domain $\Psi$ is zero), [2] gives some theoretical results on how the probability of recovering the vector $x$ successfully depend on $m$, see [2] for further details.

The compressive sensing method described above assumes that exactly $m$ out of $n$ (where $m < n$) elements of the vector $x$ are sampled. The key difficulty of using this method in wireless sensor networks is that a good amount of co-ordination is needed by the nodes to ensure that exactly $m$ elements of $x$ are sampled. In the next section, we will introduce two sampling methods such that we do not require exactly $m$ elements of $x$ are sampled, rather, we require the mean number of samples is $m$. Such probabilistic methods require less co-ordination among the nodes and are more suited for distributed implementation. Furthermore, such type of probabilistic sampling methods have not been studied and are a key contribution of this paper.

IV. NCS WITH SPARSE MATRIX

This section considers the following data recovery problem: given that a number of elements of the vector $x \in \mathbb{R}^n$ are known, the goal is to recover the unknown elements of $x$ using the knowledge that the vector $x$ is sparse in a known transform basis. Our aim is to derive the probability of recovering $x$ successfully under two different models of sampling the elements of $x$, namely uniform Bernoulli model and non-uniform Bernoulli model.

A. Problem Definition

Given an unknown vector $x$ with $n$ elements, we first sample a number of elements of $x$ and then use these sampled elements together with the fact that $x$ is sparse in a known basis to cover the unmeasured elements of $x$. In this paper, we will consider two different probability distributions for sampling the elements of $x$.

Let us first formally define the two different sampling distributions. Let $\delta_k \in \{0,1\}$ be a random variable that denotes whether the $k$-th element of $x$ is sampled, i.e. $\delta_k = 1$ if the $k$-th element of $x \in \mathbb{R}^n$ is sampled (and is therefore known), otherwise $\delta_k = 0$. In the following, $m \in [0,n]$ and $m_k \in [0,n]$ ($1 \leq k \leq n$) are parameters of the sampling distributions. We consider the following two probability distributions for $\delta_k$.

- Uniform Bernoulli model: $P[\delta_k = 1] = \frac{m}{n}$ where $P[E]$ denotes the probability that event $E$ occurs. The probability distributions of $\delta_1, \ldots, \delta_n$ are assumed to be independent.

- Non-uniform Bernoulli model: $P[\delta_k = 1] = \frac{m_k}{n}$ such that $\sum_{k=1}^n m_k = nm$. The probability distributions of $\delta_1, \ldots, \delta_n$ are assumed to be independent.

Note that in the above models, $m$, $m_1$, ..., $m_n$ are parameters chosen by the users. It is readily seen from the above definitions that the uniform Bernoulli model is a special case of the non-uniform Bernoulli model, however, we will generally assume that the $m_i$’s in the non-uniform Bernoulli model take on different values. Lastly, note that the number of sampled elements in $x$ using the above sampling models can vary from 0 to $n$, however, the average number of sampled elements is always $m$ for both models.

We assume that the vector $x$ is sparse in the transform basis $\Psi$ where the columns of $\Psi$ form the basis vectors of the transform basis. Let $\Omega$ be the set of the indices of the samples, i.e. $\Omega = \{k \in [1,n] : \delta_k = 1\}$. Let also $I \in \mathbb{R}^{n \times n}$ denote the identity matrix. We define $I_{\Omega}$ be a $|\Omega|$-by-$n$ matrix such that the $k$-th row of $I$ is also a row in $I_{\Omega}$ if $k \in \Omega$ (i.e. $\delta_k = 1$). With this definition, the vector $I_{\Omega}x \in \mathbb{R}^{|\Omega|}$ contains the sampled elements of $x$.

We will attempt to recover the unknown elements in $x$ (i.e. those elements that are not sampled) by using basis pursuit, i.e. by solving the linear programming problem shown in Eq. (2). It can be shown that both models can recover the sparse vector $x$ with high probability provided that $m$ is large enough using Eq. (2). However, the probability of recovery using the uniform model is higher than that of non-uniform model. Proof omitted due to space limit.
V. NCS APPLICATION: ENERGY-AWARE COMMUNICATIONS IN WSNs

A. Application Description and Experimental Setup

In contrast to WSNs that monitor high frequency physical phenomena (such as wind speed and wind direction), communication is the dominant energy consumption factor for WSNs that monitor low frequency physical phenomena (e.g., temperature and humidity). Table I shows the device and total energy load of a WSN that monitor relative humidity (a low frequency phenomenon) [16]. It shows that radio (communication) is indeed the dominant form of system energy load, which consumes approximately 86% of total system energy.

Table I

<table>
<thead>
<tr>
<th>Device</th>
<th>Duty cycle</th>
<th>Average current</th>
<th>The ratio of device &amp; total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensors</td>
<td>1.67%</td>
<td>9 (µA)</td>
<td>86%</td>
</tr>
<tr>
<td>Radio</td>
<td>1%</td>
<td>206 (µA)</td>
<td>86.0%</td>
</tr>
<tr>
<td>Microcontroller</td>
<td>0.4%</td>
<td>9.6 (µA)</td>
<td>4%</td>
</tr>
<tr>
<td>Quiescent</td>
<td>15 (µA)</td>
<td></td>
<td>6.2%</td>
</tr>
</tbody>
</table>

Therefore, in order to reduce network energy consumption and extend network lifetime, one should focus on how to reduce communication energy consumption in WSNs that monitor low frequency phenomena. We will demonstrate how to use the NCS framework to exploit different sensor-to-sink communication energy consumption to create a heterogeneous sampling profile among the sensor nodes. The end result is that we are able to obtain a better accuracy of spatial-temporal profile for a given energy budget. Lastly, we assume in this section that the nodes use constant transmission power.

Fig. 2 shows an indoor sensor network (54 nodes) deployed at Intel Berkeley lab to monitor low frequency phenomena (e.g., temperatures and humidity). Each sensor collected topology information and low frequency phenomena once every 31 seconds. Both topology information (bi-directional packet reception rates, see Fig. 3(a)) and sensor samples are available from the Internet1. We have chosen 986 snapshots of temperature and humidity samples, where the end-to-end data delivery rates are high (greater than 90%)2, and the topology information to evaluate NCS framework. We calculated the ETXs (the expected number of transmissions [6]) from each node to the sink using the shortest path algorithm based on the network topology information (bi-directional packet reception rates, see Fig. 3(a)), and linearly interpolated the missing samples using values from neighboring nodes. Fig. 3(a) also shows that ETXs of the nodes and it can readily be seen that the ETX for each node is different; hence this meets Assumption 2 of NCS. Intuitively, the node that consumes less energy to deliver a packet (a smaller ETX) can have a higher duty cycle.

Fig. 2. Intel Berkeley lab sensor network topology.

In Fig. 3, we studied the compressibility of the temperature and humidity signal in a number of sparsifying bases, including DCT, wavelets (Haar, Symlets) and Fourier, and found that the signal is most compressible in the DCT basis. This means that Assumption 1 of NCS is met. Therefore, both assumptions of NCS have been met in this application (e.g., a low frequency phenomenon monitoring WSN).

B. Distributed Implementation

A centralized approach typically increases the amount of communications between nodes and the base station which cost higher transmission energy. It is desirable to have a distributed implementation of NCS for energy-aware communication applications. The distributed implementation of NCS for energy-aware communication is that sensor j should sample with a probability \( g_j = \frac{m}{\sum_{j=1}^{N} 1/ETX_j} \) where \( ETX_j \) is the ETX of node j. Note that the expected number of sensors to sample at a given time instance is \( m \). Note also that \( ETX_j \) is locally available in Node j with CTP, the default multi-hop routing protocol in TinyOS. Finally, the network ETX, defined by \( \sum_{j=1}^{N} 1/ETX_j \), does not change frequently in large-scale static WSNs with wireless channels slow fading over time. A base station can then monitor the number of received samples and compare it with application desired number of samples (i.e., \( m \)). If the difference between the number of receiving samples and \( m \) is more than a threshold (e.g., 10%), the base station can send a command to adjust network ETX. Furthermore, network ETX (a four byte value) is piggybacked on the “Hello” (beacon) message of CTP, which creates little communication overhead.

C. Evaluation results

We now evaluate the performance of NCS framework in a low frequency phenomenon monitoring WSN. In particular, we are interested in the degree to which NCS framework (which we will call non-uniform Bernoulli hereafter) can reduce communication energy consumption in the WSNs compared to previously proposed uniform sampling based compressive sensing (which we call uniform Bernoulli hereafter), as well as the impact of non-uniform sampling schedule to the signal reconstruction accuracy of each nodes. We apply the non-uniform and uniform Bernoulli sampling methods to all the 986 snapshots, using one snapshot at a time.

2Although the end-to-end data delivery rates were low in 2004, recent experiment showed that high (greater than 90% or 99.9% for many experiments) end-to-end delivery rates were achievable in 12 different test-beds around the world using Collection Tree Protocol (CTP)[6].
There are a number of prior works on investigating how compressive sensing can be used to improve the efficiency of wireless sensor networks, e.g. [1], [4]. The key difference between [1], [4] is how the network acquires compressive sensing projections, which are linear combinations of the sensor readings. The paper [1] suggested to compute these projections by using an additive medium access control channel. [4] acquires projections by using message passing, and uses adaptively compressive sensing to choose the projection coefficients. A common theme of [1], [4] is to efficiently acquire projections in WSNs. Previous work assumes that sensors sample the physical phenomenon at each sampling instance but we consider non-uniform sampling in this paper, which means that some sensors do not sample at certain sampling instances.

The CS based methods discussed earlier use a dense projection matrix, which requires all the data points a signal vector to be collected. However, since only a few projections need to be transmitted, the proposed methods could save the transmission energy. Our CS implementation is different from the earlier methods since we use a sparse projection matrix and thus do not require to collect all the data points of the signal vector. A CS based data gathering approach is presented in [13] which investigates the impact of a routing topology generated sparse projection matrix on the accuracy of the approximation. Our work is different from theirs since our projection matrix is not based on the routing topology rather it is populated based on the energy profile of the sensors.

Different techniques apart from CS have been used in the past to enable adaptive sensing exploiting the temporal, spatial or spatial-temporal [11], [19] correlation of the signal. Though both our sensing/transmission approaches exploit the temporal/spatial correlation of the data points, we have considered non-uniform energy profile of the sensors, which is different from the existing literature.

Communication overhead heterogeneity has also been exploited to extend sensor network lifetime in [9], [8]. However, [9] is designed to satisfy an application’s acceptable tolerance of aggregation queries (such as min, max, sum, mean) with imprecise and inaccurate samples. On the other hand, NCS is designed to recover the whole signals with some approximation errors. The authors assume there are heterogenous sensor
nodes, with heterogeneous radios and initial energy supplies, in the networks [8]. NCS can work with sensor networks that have homogenous radios and initial energy supplies.

VII. CONCLUSIONS

In this paper, we consider a large scale wireless sensor network (WSN) measuring spatio-temporal correlated physical phenomena, i.e., compressible signals. At the same time, we also consider that there is heterogeneity in resource consumption in the WSNs, which is a common phenomenon. We proposed a framework to address the problem of heterogeneous sensor sample schedule (non-uniform sampling) signal reconstruction by extending Compressive Sensing (CS) theory, to exploit the heterogeneity in a WSN that monitor compressible signals, in order to improve network performance. We evaluated proposed NCS extensively with a real WSN application dataset, and presented a distributed implementation of NCS framework that introduced very little communication overhead, and show that, compared to previously proposed approaches based on traditional CS NCS achieved similar signal approximation accuracy with significantly less samples (energy consumption).

REFERENCES


