

### Neural Attributed Community Search at Billion Scale

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### Outline

- Background and Problem Definition
- Motivations
- Methods
- Experiments
- Summary

# Background

#### • Graph is everywhere.







Cora (citation graph) [1]

Social graph [2]

Nodes are often featured with attributes

• **Community:** Normally, a set of nodes that are densely connected internally and loosely connected externally.

### Problem definition

• Attributed Community Search: Given an attributed graph G(V, E, F), and a query  $q = \langle V_q, F_q \rangle$  where  $V_q \subseteq V$  is a set of query nodes and  $F_q = \subseteq F$  is a set of query attributes, the task of attributed community search (ACS) aims to find a query-dependent community which preserves both structure cohesiveness and attribute homogeneity.

#### Applications

Research communities mining.

Friend recommendation.

Protein complex identification.



### Motivation



• Existing learning-based methods:



### Our methods



#### Candidate Subgraph Extraction

Structure-based pruning with density sketch modularity

Attribute-based pruning

• Consistency-aware Net (CoNet):

Cross-Attention Encoder

Structure-Attribute Consistency & Local Consistency

## Density sketch modularity

Graph Modularity is a widely used measure for community cohesiveness. A higher modularity indicates a more cohesive community



## Analysis of density sketch modularity

- When employing classic modularity for CS, it suffers from the free-rider effect and the resolution limit problem
- Free-rider effect

Given a set of query q, let C be a community identified based on a goodness function f, and  $C^*$  be the optimal solution (either local or global). The goodness function is said to be affected by the free-rider effect if  $f(C \cup C^*) \ge f(C)$ .

Resulting community may encompass numerous nodes unrelated to the query nodes

#### Resolution limit problem

Given a graph G, query q, the objective function f, a community constraint T, a community C satisfying T and containing all the query q, and any community C'satisfying the constraint T such that  $C \cup C'$  is connected and  $C \cap C' = \emptyset$ , the objective function is said to suffer from the resolution limit problem if there exists a community C'such that  $C \cup C'$  satisfies the constraint T and  $f(C \cup C') \ge f(C)$ .



Resultant community may be too large to highlight some important structures.

### Analysis of density sketch modularity

For any positive  $\tau$ , whenever density sketch modularity suffers from the free-rider effect, classic modularity suffers from the free-rider effect as well.

$$\begin{split} DSM(G, C \cup C*) &\geq DSM(G, C) \\ \Rightarrow \frac{1}{2|V_{C \cup C^*}|^{\tau}} (2|E_{C \cup C^*}| - \frac{d_{C \cup C^*}^2}{2|E|}) \geq \frac{1}{2|V_C|^{\tau}} (2|E_C| - \frac{d_C^2}{2|E|}) \\ \operatorname{As} 2|V_C|^{\tau} &> 0 \\ \Rightarrow \{\frac{|V_C|}{|V_{C \cup C^*}|}\}^{\tau} (2|E_{C \cup C^*}| - \frac{d_{C \cup C^*}^2}{2|E|}) \geq 2|E_C| - \frac{d_C^2}{2|E|} \\ \Rightarrow 2|E_{C \cup C^*}| - \frac{d_{C \cup C^*}^2}{2|E|} \geq \{\frac{|V_C|}{|V_{C \cup C^*}|}\}^{\tau} (2|E_{C \cup C^*}| - \frac{d_{C \cup C^*}^2}{2|E|}) \geq 2|E_C| - \frac{d_C^2}{2|E|} \\ \Rightarrow CM(G, C \cup C^*) \geq CM(G, C) \end{split}$$

For any positive  $\tau$ , whenever density sketch modularity suffers from the resolution-limit problem, classic modularity suffers from the resolution-limit problem as well.



### Candidate subgraph extraction

Structure-based pruning

 $\checkmark k$ -hop neighborhood with largest density sketch modularity (adaptively)



- >1-hop DSM: 0.504
  >2-hop DSM: 0.507
  >3-hop DSM: 0.135
  >3
  - ≻1-hop DSM: 0.504
  - ≻2-hop DSM: -0.094
  - ≻3-hop DSM: 0.0

• Attribute-based pruning:



Figure 4: node-attribute bipartite graph

*k*-hop neighborhood with largest bipartite modularity in the node-attribute bipartite graph

### CoNet architecture





Figure 6: Illustration of Cross Attention Encoder

$$\bigvee \text{ Query Encoding } X_q = H_{v_q}^{(k)} W_q^{(s,k)}, X_k = H^{(s,k)} W_k^{(s,k)}, X_v = H^{(s,k)} W_v^{(s,k)}$$
$$X = \text{softmax}(\frac{X_q X_k^T}{\sqrt{d_{k+1}}}), H_{v_q}^{(k+1)} = X X_v$$
$$\text{ Graph Encoding } h_v^{(s,k+1)} = \text{ML}P^{(s,k)} \left( \left(1 + \epsilon^{(k)}\right) \cdot h_v^{(s,k)}, \sum_{v' \in N(v)} h_v \prime^{(s,k)} \right)$$

Lemma: ConNet is as powerful as the 1-WL algorithm.

### Training Objectives

- Structure-Attribute Consistency
  - $\checkmark$

Minimize the Wasserstein-1 distance between structure distribution and attribute distribution

$$egin{aligned} W_1(\mathbb{P}_s,\mathbb{P}_a)&=\inf_{\gamma\in\pi(\mathbb{P}_s,\mathbb{P}_a)}\mathbb{E}_{(\mu,
u)\sim\gamma}[||\mu-
u||]\ W_1(\mathbb{P}_s,\mathbb{P}_a)&=\sup_{||f_w||_L\leq 1}\mathbb{E}_{\mu\sim\mathbb{P}_s}[f_w(\mu)]-\mathbb{E}_{
u\sim\mathbb{P}_a}[f_w(
u)] \end{aligned}$$

$$\mathcal{L}_w(H^{(s)}, H^{(a)}) = \sum_{h_v^{(a)} \in H^{(a)}} f_w(h_v^{(a)}) - \sum_{h_u^{(s)} \in H^{(s)}} f_w(h_u^{(s)})$$
presistoncy

$$\mathcal{L} = \mathcal{L}_b + lpha \mathcal{L}_w + eta \mathcal{L}_m$$

• Local Consistency

 $\checkmark$  Neighboring nodes have similar prediction

$$\mathcal{L}_m(H,A) = \left| \left| A - H H^T 
ight| 
ight|_F$$

• Ground-truth information

$$\mathcal{L}_b( ilde{C}_{q_1}, C_{q_i}) = \sum_{i=1}^{|V_{sub}|} - C_{q_i,j} log( ilde{C}_{q_1,j}) + (1-C_{q_i,j}) log(1- ilde{C}_{q_1,j})$$

### Experiments

#### **Table 2: Statistics of the datasets**

Dataset	V	E	$ F^d $	N <sub>c</sub>
Texas	187	279	1703	5
Cornell	195	285	1703	5
Washt	230	392	1703	5
Wiscs	265	469	1703	5
Cora	2708	5429	1433	7
Citeseer	3312	4715	3703	6
Google+	7856	321,268	2024	91
PubMed	19,717	44,324	500	3
Reddit	232,965	47,396,905	602	41
Orkut	3,072,627	117,185,083	1000	5000
Friendster	65,608,366	1,806,067,135	1000	5000

### • Query settings

- Attribute from communities (AFC)
  Attribute from query node (AFN)
  Empty attribute query (EmA)
- Metrics
  - F1-score
  - Average degree (Avg.d)
  - Community pair-wise Jaccard (CPJ)

Experiments



Learning-based method has an average improvement of 54.50% in F1-score compared with traditional ACS method.

ALICE has an average improvement of 10.18% compared to SOTA AQD-GNN using AFN as the query attribute.

### Experiments

Method	Texas	Cornell	Washt	Wisc	Cora	Citeseer	Google+	Pubmed	Reddit	Orkut	Frienster
ICS-GNN (Train)	***	***	***	***	***	***	***	***	***	***	***
AQD-GNN (Train)	2.2+233	2.1+234	2.5+239	2.9+232	64.1+2214	59.3+4390	834.6+10035	3171.8+37059	—	—	-
ALICE (Train)	2.6+344	2.5+381	3.8+332	1.8 + 324	16.32+509	59.8+1239	189.8+3256	123.5+4317	8681+1107	2594.8+2224	65415.6+1244
ICS-GNN (Query)	20.5	25.1	27.4	28.6	167.7	124.3	627.6	112.3	1034.7	1540.8	24253.7
AQD-GNN (Query)	0.015 + 0.0021	0.014+0.0020	0.017 + 0.0022	0.019 + 0.0020	0.427+0.0026	0.395+0.0019	5.564+0.0019	21.14+0.0019	_	_	_
ALICE (Query)	0.017+0.0053	0.017+0.0045	0.025 + 0.0044	0.014 +0.0050	0.104+0.0041	0.398+0.0047	1.26+0.0053	0.823+0.0058	5.78+0.0052	17.29+0.0045	436.1+0.0048

#### Table 3: Efficiency evaluation on different datasets (in seconds)

(1): We report preparation time + train (query) time; (2): - indicates out of memory or not finished within 7 days; (3): \*\*\* indicates this cell not applicable to this model.





### Summary

• We propose a learning-based framework, named ALICE, for attributed community search at large scale.

• We design an efficient subgraph extraction algorithm by leveraging density sketch modularity and node-attribute relationship to adaptively select promising nodes.

• We propose a GNN-based model ConNet to preserve both structure-attribute consistency and local consistency among nodes.

• Extensive experiments over 11 popular public datasets, encompassing one billion-scale graph Friendster, demonstrate the effectiveness of ALICE.

