COMP1927 16x1 Computing 2

Complexity

Problems, Algorithms, Programs and Processes

- Problem: A problem that needs to be solved
- Algorithm: Well defined instructions for completing the problem
- Program: Implementation of the algorithm in a particular programming language
- Process: An instance of the program as it is being executed on a particular machine

Analysis of software

- What makes software "good"?
  - returns expected result for all valid inputs
  - behaves "sensibly" for non-valid inputs
  - clear code, easy to maintain/modify
  - interface is clear and consistent (API or GUI)
  - returns results quickly (even for large inputs)

    Ie. It is efficient
- We may sometimes also be interested in other measures
  - memory/disk space, network traffic, disk IO etc

Algorithm Efficiency

- The algorithm is by far the most important determinant of the efficiency of a program
- Small speed ups in terms of operating systems, compilers, computers and implementation details are irrelevant
- May give small speed ups but usually only by a small constant factor

Determining Algorithm Efficiency

- At the design stage
  - Theoretical approach
  - complexity theory
- After the testing stage
  - Once it is implemented and correct you can empirically evaluate performance eg using the time command

Timing

- Note we are not interested in the absolute time it takes to run.
- We are interested in the relative time it takes as the problem increases
- Absolute times differ on different machines and with different languages

Complexity Theory Example

```
int linearSearch(int a[], int n, int key){
    for indexes from 0 to n-1
        if key equals current element array
            return current index
    return -1
}
```

- What is the worst case cost?
  - When does this occur?
  - How many comparisons between data instances were made?

Complexity Example

- How many times does each line run in the worst case?

C0: line 2: For loop n+1 times

C1: line 3: n comparisons

C2: line 4: 0 times (worst case)

C3: line 5: 1 time (worst case)

Total: C0(n+1) + C1(n) + C3 = O(n)

- For an unsorted sequence that is the best we can do

Informal Definition of Big-O Notation

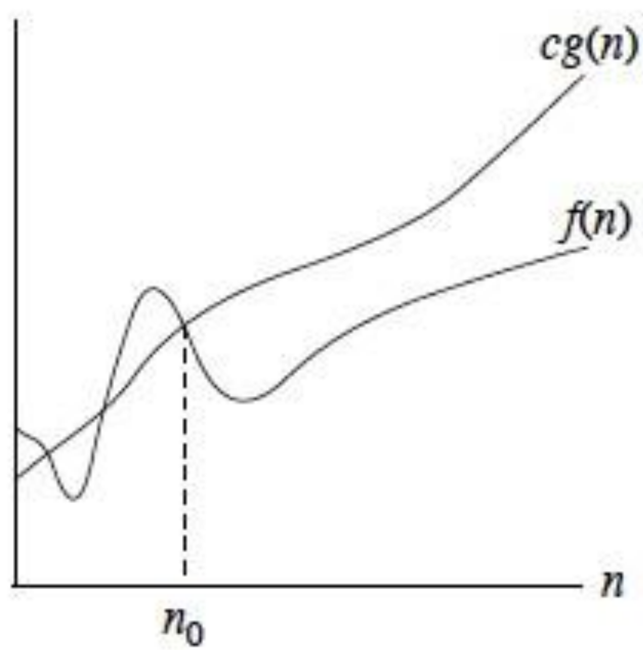
- We express complexity using **big-O notation**
- Represents the asymptotic **worst case** (unless stated otherwise) time complexity

- Big-O expressions do not have constants or low-order terms as when n gets larger these do not matter
- For example: For a problem of size n, if the cost of the worst case is
  - $1.5n^2 + 3n + 10$
  - in Big-O notation would be $O(n^2)$

Big O-notation Formal Definition

The big O-notation is used to classify the work complexity of algorithms

Definition: A function $f(n)$ is said to be in (the set) O($g(n)$) if there exist constants c and N_o such that $f(n) < c * g(n)$ for all $n > N_o$



Empirical Analysis Linear Search

- Use the ‘time’ command in linux.

Run on different sized inputs

```
time ./prog < input > /dev/null
```

not interested in real-time

interested in user-time

What is the relationship between

- input size
- time

Size of input(n)	Time
100000	
1000000	
10000000	
100000000	

Predicting Time

- If I know my algorithm is quadratic and I know that it takes 1.2 seconds to run on a data set of size 1000
- Approximately how long would you expect to wait for a data set of size 2000?
- What about 10000?
- What about 100000?
- What about 1000000?
- What about 10000000?

Searching in a Sorted Array

- Given an array a of N elements, with a[i] <= a[j] for any pair of indices i,j, with i <= j < N,
- search for an element e in the array

```
int a[N];        // array with N items
int found = 0;
int i = 0;

while ((i < N) && (!found)){
    found = (a[i] == e);
                    i++;
 }
```

Searching in a Sorted Array

- Given an array a of N elements, with a[i] <= a[j] for any pair of indices i,j, with i <= j < N,
- search for an element e in the array

```
int a[N];        // array with N items
int found = 0;
int finished = 0;
int i = 0;
while ((i < N) && (!found) && (!finished)){
    found = (a[i] == e);
    finished = (e < a[i]);
    i++;
 }
```

exploit the fact that a is sorted

Searching in a Sorted Array

- How many steps are required to search an array of N elements

  Best case: $T_N = 1$

  Worst case: $T_N = N$

  Average: $T_N = N/2$

- Still a linear algorithm, like searching in a unsorted array

Binary Search

- We start in the middle of the array:
- if a[N/2] == e, we found the element and we’re done
- and, if necessary, `split’ array in half to continue search
- if a[N/2] < e, continue search on a[0] to a[N/2 -1]
- if a[N/2] > e, continue search on a[N/2+1] to a[N-1]
- This algorithm is called binary search.

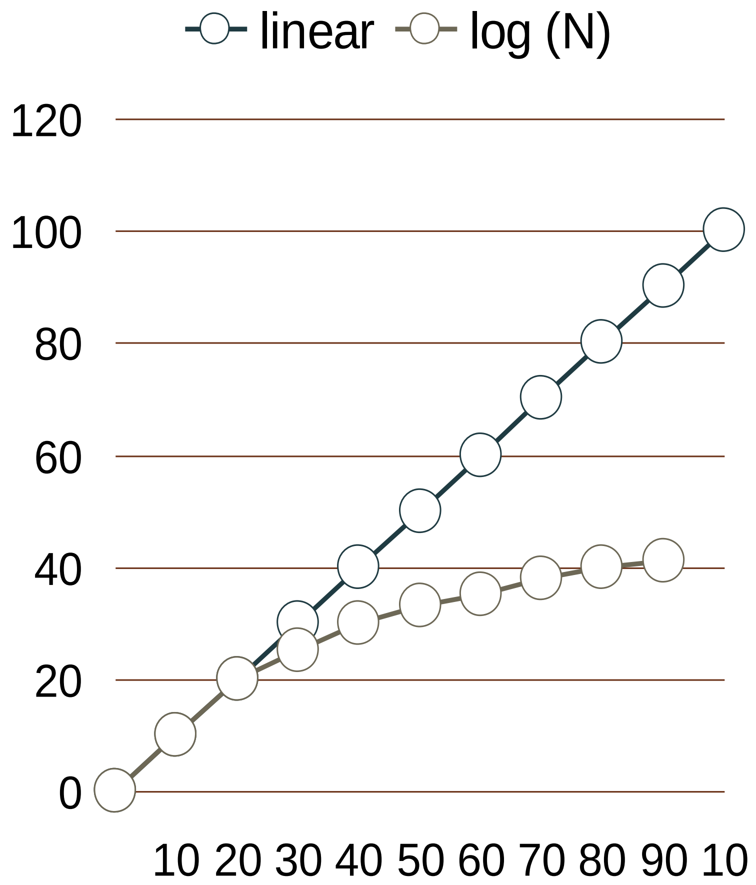
Binary Search

See binary.c for implementation

- We maintain two indices, l and r, to denote leftmost and rightmost array index of current part of the array
  - initially l=0 and r=N-1
- iteration stops when:
  - left and right index define an empty array, element not found
  - Eg l > r
  - a[(l+r)/2] holds the element we're looking for
- if: a[(l+r)/2] is larger than element, continue search on left
  a[l]..a[(l+r)/2-1]
  else continue search on right
  a[(l+r)/2+1]..a[r]

Binary Search

- How many comparisons do we need for
- an array of size N?
  - Best case:
    - $T_N = 1$
  - Worst case:
    - $T_1 = 1$
    - $T_N = 1 + T_{N/2}$
    - $T_N = \log_2 N + 1$
    - $O(\log n)$
- Binary search is a
  - logarithmic algorithm



Big-O Notation

- All constant functions are in O(1)
- All linear functions are in O(n)
- All logarithmic function are in the same class O($log(n)$)
  - O($log_2(n)$) = O($log_3(n)$)= ....
    - (since $log_b(a) * log_a(n) = log_b(n)$)
- We say an algorithm is O$(g(n))$ if, for an input of size n, the algorithm requires $T(n)$ steps, with $T(n)$ in O$(g(n))$, and O($g(n)$) minimal
  - binary search is $O(log(n))$
  - linear search is O(n)
- We say a problem is O($g(n)$) if the best algorithm is O($g(n)$)
  - finding the maximum in an unsorted sequence is O(n)

Common Categories

- $O(1)$: constant - instructions in the program are executed a fixed number of times, independent of the size of the input
- $O(\log N)$: logarithmic - some divide & conquer algorithms with trivial splitting and combining operations
- $O(N)$: linear - every element of the input has to be processed, usually in a straight forward way
- $O(N * \log N)$: Divide &Conquer algorithms where splitting or combining operation is proportional to the input
- $O(N^2)$: quadratic. Algorithms which have to compare each input value with every other input value. Problematic for large input
- $O(N^3)$: cubic, only feasible for very small problem sizes
- $O(2^N)$: exponential, of almost no practical use

Complexity Matters

n	**log n**	**nlogn**	**n^2**	**2^n**
10	4	40	100	1024
100	7	700	10000	1.3E+30
1000	10	10000	1000000	REALLY BIG
10000	14	140000	100000000	
100000	17	1700000	10000000000	
1000000	20	20000000	1000000000000	

Exercise

What would be the time complexity of inserting an element at the beginning of

- a linked list
- an array

What would be the time complexity of inserting an element at the end of

- a linked list
- an array