Recursion

COMP1927 16x1

Sedgewick Chapter 5

Recursive Functions

- problems can sometimes be expressed in terms of a simpler instance of the same problem
- Example: factorial
  - 1! = 1
  - 2! = 1 * 2
  - ...
  - (N-1)! = 1 * 2 * 3 * ... * (N-1)
  - N! = 1 * 2 * 3 * ... * (N-1) * N

2! = 1! * 2

N! = (N-1)! * N

Recursive Functions

- Solving problems recursively in a program involves
  - Developing a function that calls itself
  - Must include
    - Base Case: aka stopping case: so easy no recursive call is needed
    - Recursive Case: calls the function on a ‘smaller’ version of the problem

Iteration vs Recursion

- Compute `N! = 1 * 2 * 3 * .... * N`

```
//An iterative solution
int factorial(int N){
    result = 1;
    for (i = 1; i <= N; i++)
        result = i * result;
    return result;
}
```

- Alternative Solution: factorial calls itself recursively

```
int factorial (int N) {
  if (N == 1) {
    return 1;
  } else {
    return N * factorial (N-1);
  }
}
```

base case

recursive case

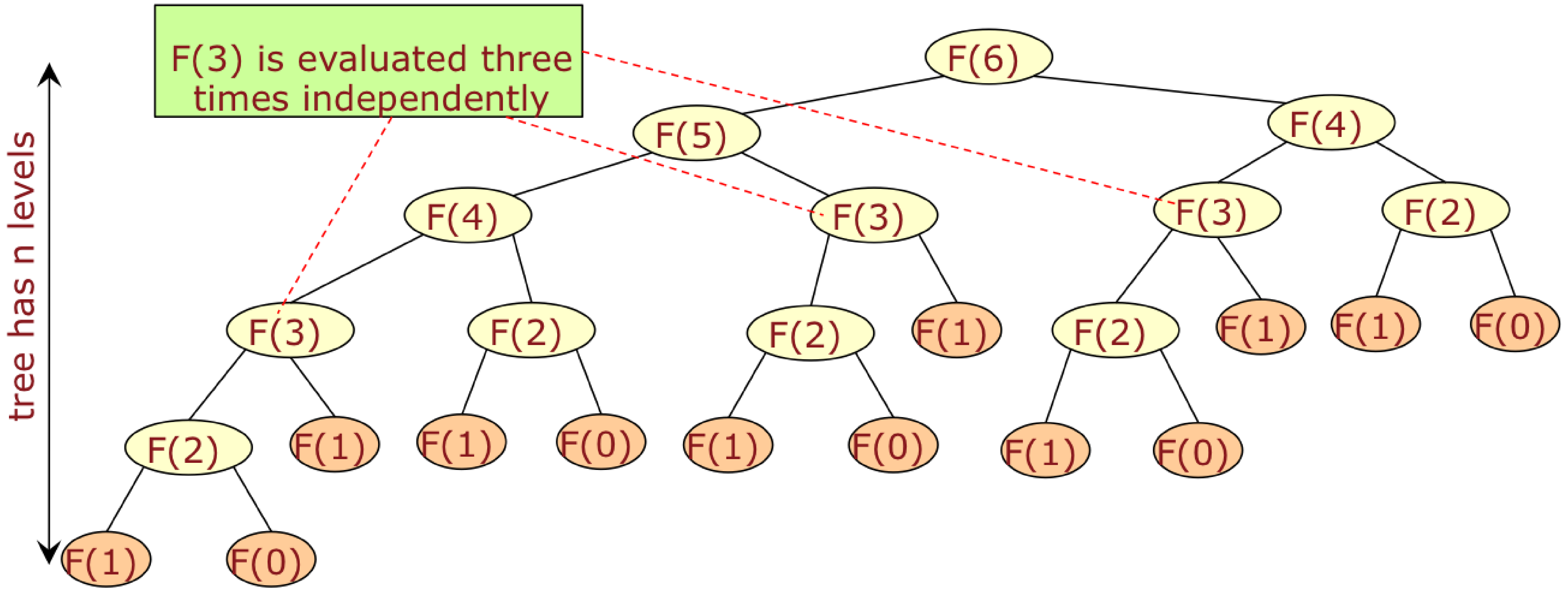
Bad Fibonacci

- Sometimes recursive code results in horribly in-efficient code that re-evaluates things over and over.
- 2^n calls: $O(k^n)$ - exponential
- Exponential functions can only be used in practice for very small values of n

```
//Code to return the nth fibonacci number
//0 1 1 2 3 5 8 13 21
int badFib(int n){
      if(n == 0) return 0;
      if(n == 1) return 1;
      return badFib(n-1) + badFib(n-2);
}
```

Why badFib is bad

- Tracing calls on BadFib produces a tree of calls where intermediate results are recalculated again and again.



Linked Lists

A linked list can be described recursively

- A list is comprised of a
  - head (a node)
  - a tail (the rest of the list)

```
typedef struct node * link;

struct node{
    int item;
    link next;
};
```

Recursive List Functions

- We can define some list operations as recursive functions:
  - length: return the length of a list
  - sumOfElems: return the length of a list
  - printList: print the list
  - printListReverse: print out the list in reverse order
- Recursive list operations are not useful for huge lists
  - The depth of recursion may be proportional to the length of the list

Recursive List Functions

```
int length (link ls) {
  if (ls == NULL) {
    return 0;
  }
  return 1 + length (ls->next);
}
```

base case

recursive case

```
int sumOfElems (link ls) {

    if (ls == NULL) {
        return 0;

    }
    return (ls->item + sumOfElems(ls->next));
}
```

base case

recursive case

Recursive List Functions

```
void printList(link ls){
    if(ls != NULL){
        printf(“%d\n“,ls->item);
        printList(ls->next);
    }
}
```

```
//To print in reverse change the
//order of the recursive call and
//the printf
void printListReverse(link ls){
    if(ls != NULL){
        printListReverse(ls->next);
        printf(“%d\n“,ls->item);
    }
}
```

Divide and Conquer

Basic Idea:

- divide the input into two parts
- solve the problems recursively on both parts
- combine the results on the two halves into an overall solution

Divide and Conquer

Divide and Conquer Approach for finding maximum in an unsorted array:

- Divide array in two halves in each recursive step

  Base case

  - subarray with exactly one element: return it

  Recursive case

  - split array into two
  - find maximum of each half
  - return maximum of the two sub-solutions

Iterative solution

```
//iterative solution O(n)
int maximum(int a[], int n){
    int a[N];
    int max = a[0];
    int i;
    for (i=0; i < n; i++){
        if (a[i] > max){
            max = a[i];
        }
    }
    return max;
}
```

Divide and Conquer Solution

```
//Divide and conquer recursive solution
int max (int a[], int l, int r) {
    int m1, m2;
    int m = (l+r)/2;
    if (l==r) {
        return a[l];
    }
    //find max of left half
    m1 = max (a,l,m);
    //find max of right half
    m2 = max (a, m+1, r)
    //combine results to get max of both halves
    if (m1 < m2) {
        return m2;
    } else {
        return m1;
    }
}
```

Complexity Analysis

How many calls of max are necessary for the divide and conquer maximum algorithm?

- Length = 1

  $T_1 = 1$

- Length = $N > 1$

  $T_N = T_{N/2} + T_{N/2} + 1$

- Overall, we have

  $T_N = N + 1$

  In each recursive call, we have to do a fixed number of steps (independent of the size of the argument)

- O(N)

Recursive Binary Search

Maintain two indices, l and r, to denote leftmost and rightmost array index of current part of the array

- initially l=0 and r=N-1

Base cases:

- array is empty, element not found
- a[(l+r)/2] holds the element we're looking for

Recursive cases: a[(l+r)/2] is

- larger than element, continue search on a[l]..a[(l+r)/2-1]
- smaller than element, continue search on a[(l+r)/2+1]..a[r]

O(log(n))