Graph Algorithms and Computability

Computing 2 COMP1927 16x1
Hamilton path:
  - is there a simple path connecting two vertices that visits each vertex in the graph exactly once?

Hamilton tour:
  - is there a cycle in the graph that visits each vertex exactly once?

Named after the Irish mathematician, physicist and astronomer Sir William Rowan Hamilton (1805 - 1865)
Brute force search: we can adapt the simple path search to look for a Hamilton path:

- keep a counter of vertices visited in the current path
- only accept a path if the counter indicates that it contains all vertices
HAMilton Path

- For simple paths we know that
  - if there is no simple path from $t$ to $w$, then there is no simple path from $v$ to $w$ via $t$
  - so, there is no point visiting a vertex twice in the algorithm

- Unfortunately, this is not true for Hamilton paths
  - we have to inspect every possible path in the graph!

- What does this mean for the number of recursive calls necessary to find a Hamilton path?
  - in a complete graph, we have $V!$ different paths ($\approx (V/e)^V$)

- Finding whether there is a Hamilton Path in a graph is an $NP$-complete problem
**NP (Non-deterministic Polynomial) Class of Problems**

- A problem is in the class \( NP \), if it is a decision problem and the correctness of its answer can be checked in polynomial time.
- A problem is in the class \( P \), if it is a decision problem and its answer can be computed in polynomial time.
- A problem is \( NP \) complete, if it is in \( NP \) and at least as difficult as the most difficult problem in \( NP \).
- No polynomial algorithms are known for these problems.
- Examples of \( NP \) complete problems:
  - Hamilton path problem
  - Travelling salesman problem
  - Knapsack problem
Euler Path

- Is there a path in the graph connecting two vertices that uses each edge in the graph exactly once?
  - vertices can be visited any number of times
- If the path is from a vertex back to itself it is called an Euler tour
- Named after the Swiss mathematician and physicist Leonard Euler (1707-1783):
  - is there a way to cross all the bridges of Königsberg exactly once on a walk through the town?
**Euler Path**

- Naive recursive algorithm would result in factorial time performance
- Euler path problem turns out to be *much easier* than Hamilton Path
  - $O(E+V)$ adjacency list
  - $O(V^2)$ adjacency matrix
- A graph has an Euler tour if and only if
  - it is connected, and
  - all vertices are of even degree
- A graph has an Euler path if and only if
  - it is connected, and
  - exactly two of its vertices are of odd degree