Graph Algorithms and Computability

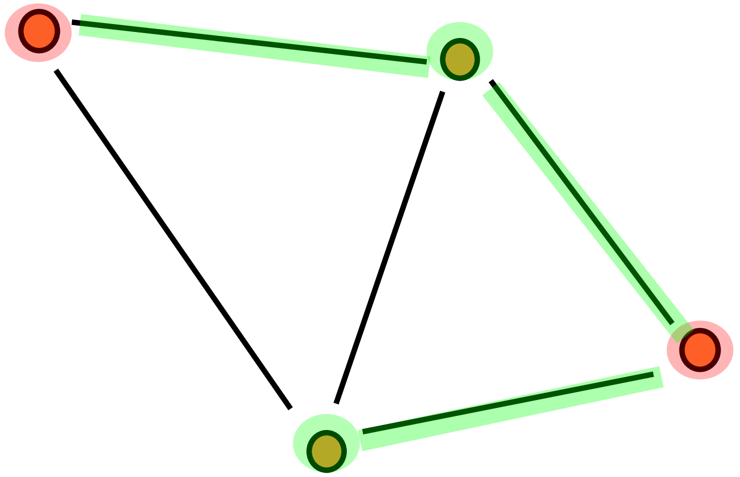
Computing 2 COMP1927 16x1

Hamilton Path

- Hamilton path:
  - is there a simple path connecting two vertices that visits each vertex in the graph exactly once?
- Hamilton tour:
  - is there a cycle in the graph that visits each vertex exactly once?
- Named after the Irish mathematician, physicist and astronomer Sir William Rowan Hamilton (1805 - 1865)

HAMILTON PATH

- Brute force search: we can adapt the simple path search to look for a Hamilton path:
  - keep a counter of vertices visited in the current path
  - only accept a path if the counter indicates that it contains all vertices

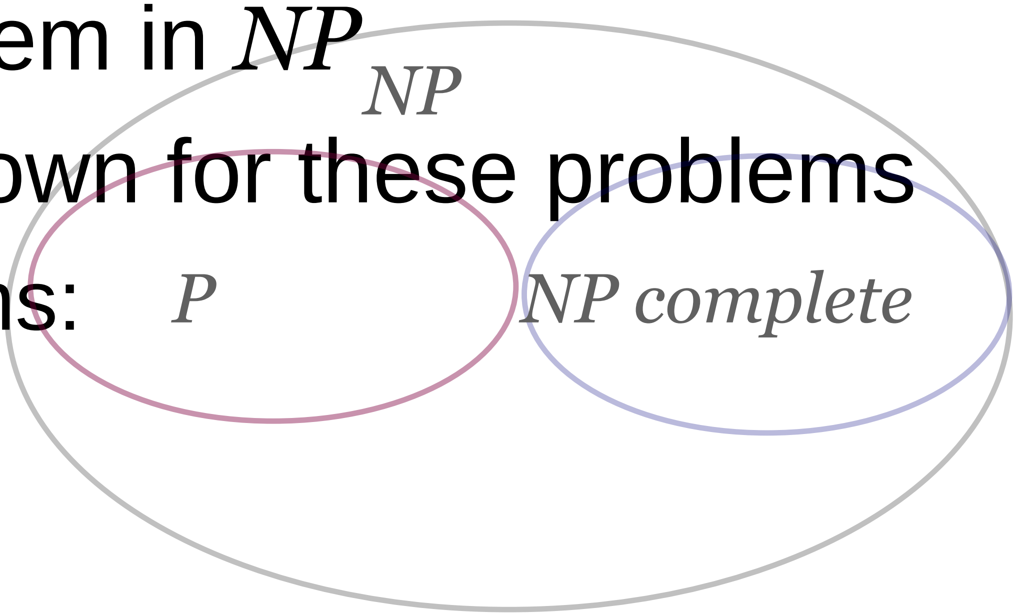


HAMILTON PATH

- For simple paths we know that
  - if there is no simple path from t to w, then there is no simple path from v to w via t
  - so, there is no point visiting a vertex twice in the algorithm
- Unfortunately, this is not true for Hamilton paths
  - we have to inspect every possible path in the graph!
- What does this mean for the number of recursive calls necessary to find a Hamilton path?
  - in a complete graph, we have $V!$ different paths ($\approx (V/e)^V$)
- Finding whether there is a Hamilton Path in a graph is an NP-complete problem

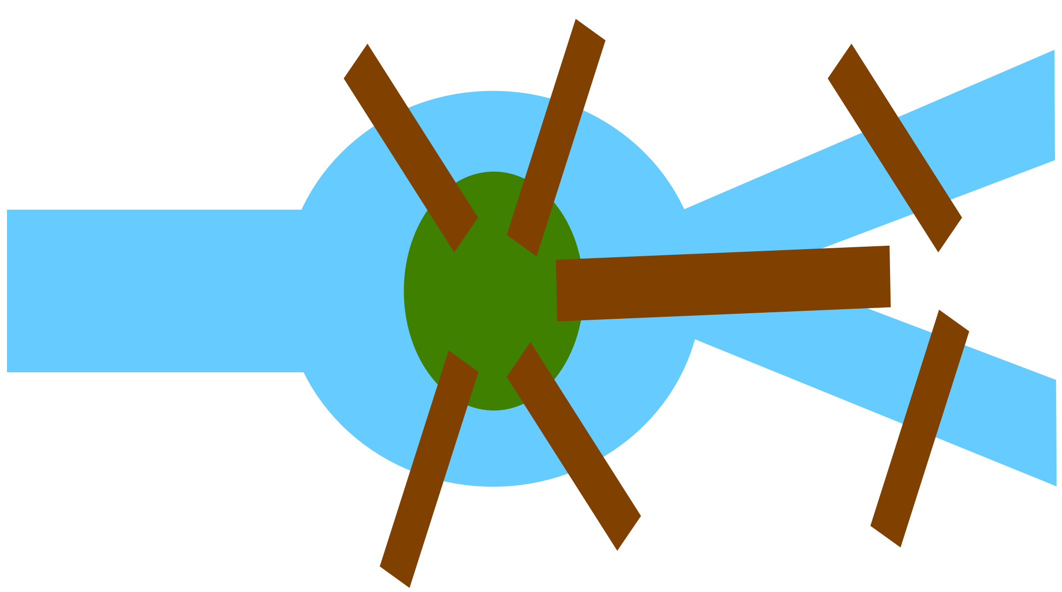
NP (Non-deterministic Polynomial) Class of Problems

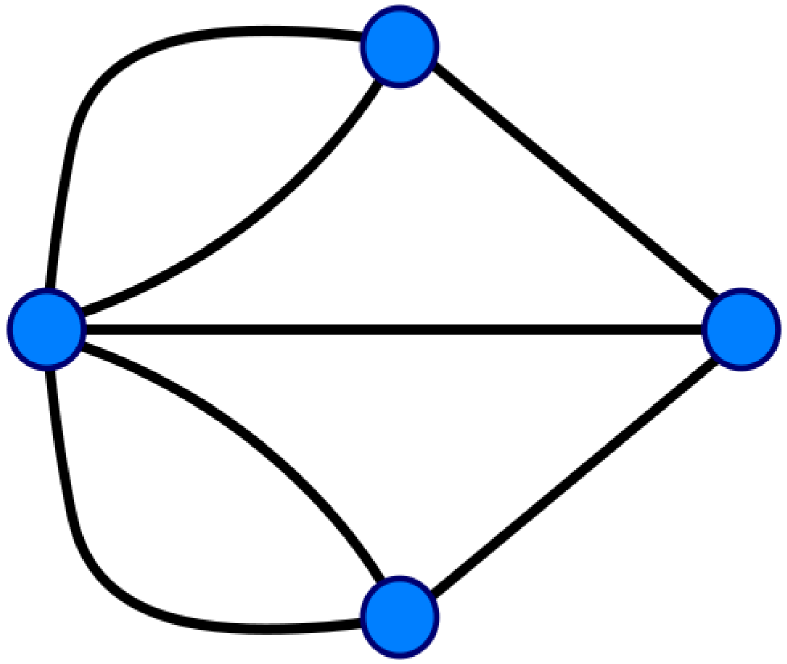
- A problem is in the class *NP*, if it is a decision problem and the correctness of its answer can be checked in polynomial time
- A problem is in the class *P*, if it is a decision problem and its answer can be computed in polynomial time
- A problem is *NP* complete, if it is in *NP* and at least as difficult as the most difficult problem in *NP*
- No polynomial algorithms are known for these problems
- Examples of *NP* complete problems:
  - Hamilton path problem
  - Travelling salesman problem
  - Knapsack problem



EULER PATH

- Is there a path in the graph connecting two vertices that uses each edge in the graph exactly once?
    - vertices can be visited any number of times
- If the path is from a vertex back to itself it is called an Euler tour
- Named after the Swiss mathematician and physicist Leonard Euler (1707-1783):
    - is there a way to cross all the bridges of Königsberg exactly once on a walk through the town?





Euler Path

- Naive recursive algorithm would result in factorial time performance
- Euler path problem turns out to be much easier than Hamilton Path
    - O(E+V) adjacency list
    - O(V^2) adjacency matrix
- A graph has an Euler tour if and only if
    - it is connected, and
    - all vertices are of even degree
- A graph has an Euler path if and only if
    - it is connected, and
    - exactly two of its vertices are of odd degree