Priority Queues and Heaps

Computing 2 COMP1927 17x1

Priority

Some applications of queues require items processed in order of "key" or priority rather than in order of entry (FIFO)

Priority Queues (PQueues or PQs) provide this via:

- Insert item with a given priority into PQ
- Remove item with highest priority key
  - Highest priority key may be one with smallest or largest value depending on the application
- Plus generic ADT operations:
  - new, drop, empty, …

Priority Queue Interface

```
typedef struct priQ * PriQ;
//We assume we have a more complex Item type that has
//a key and a value, where the key is the priority and the
//value is the data being stored

// Core operations
PriQ initPriQ(void);
void insert(PriQ q, Item i);
//retrieve and delete Item with highest priority
Item delete(PriQ q);

// Useful operations
int sizePriQ(PriQ q);
void changePriority(PriQ q, Key k, Item i);
void deleteKey(PriQ q, Key k);
int maxSize(PriQ q);
```

Comparison of Priority Queue Implementations

Data Structure	Insert	Delete	IsEmpty
Sorted Array	*O(N)*	*O(1)*	*O(1)*
Unsorted Array	*O(1)*	*O(N)*	*O(1)*
Sorted List	*O(N)*	*O(1)*	*O(1)*
Unsorted List	*O(1)*	*O(N)*	*O(1)*

Can we implement BOTH operations efficiently?

Yes with a heap

O(log N) for insert and delete

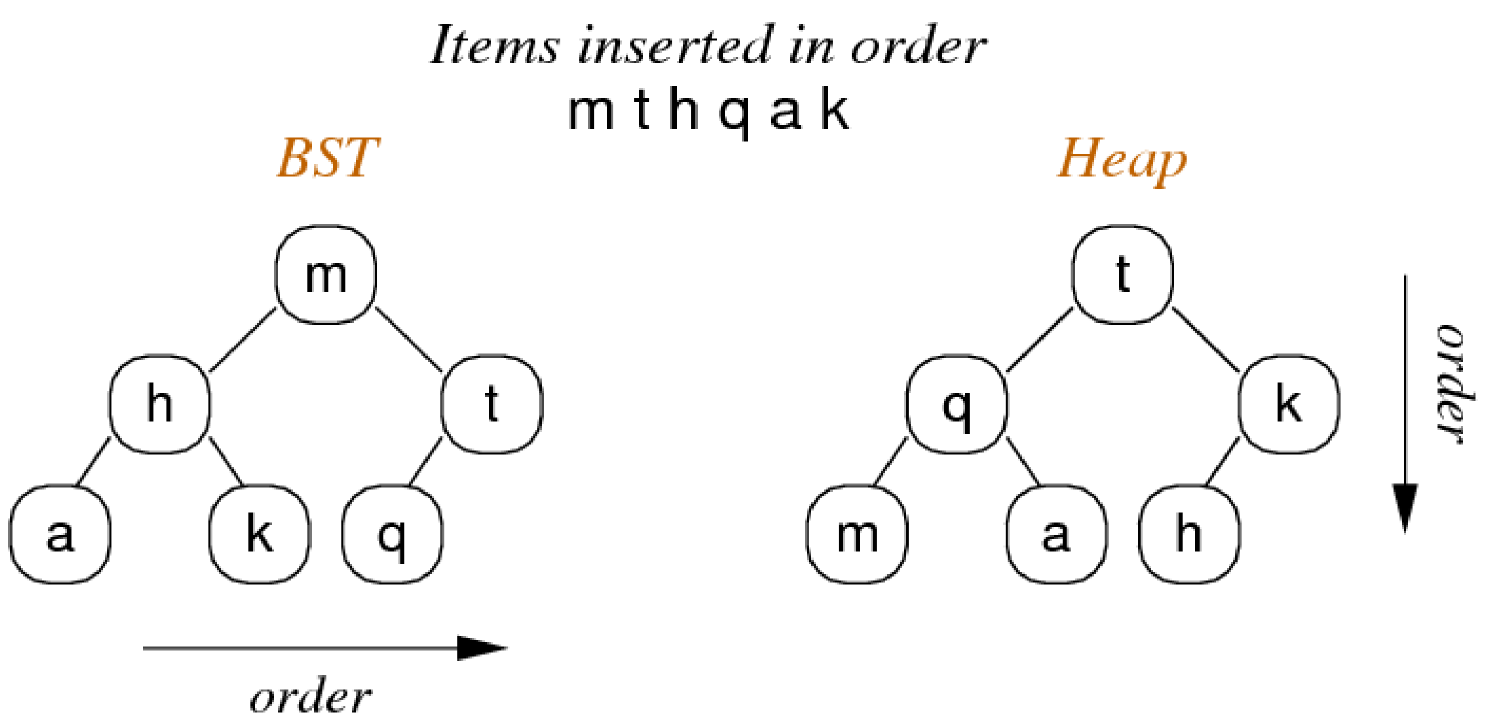
Heap	*O(logN)*	*O(logN)*	*O(1)*

Heap Order property

Heaps can be viewed as trees with top-to-bottom heap ordering

- for all keys both subtrees are ≤ root

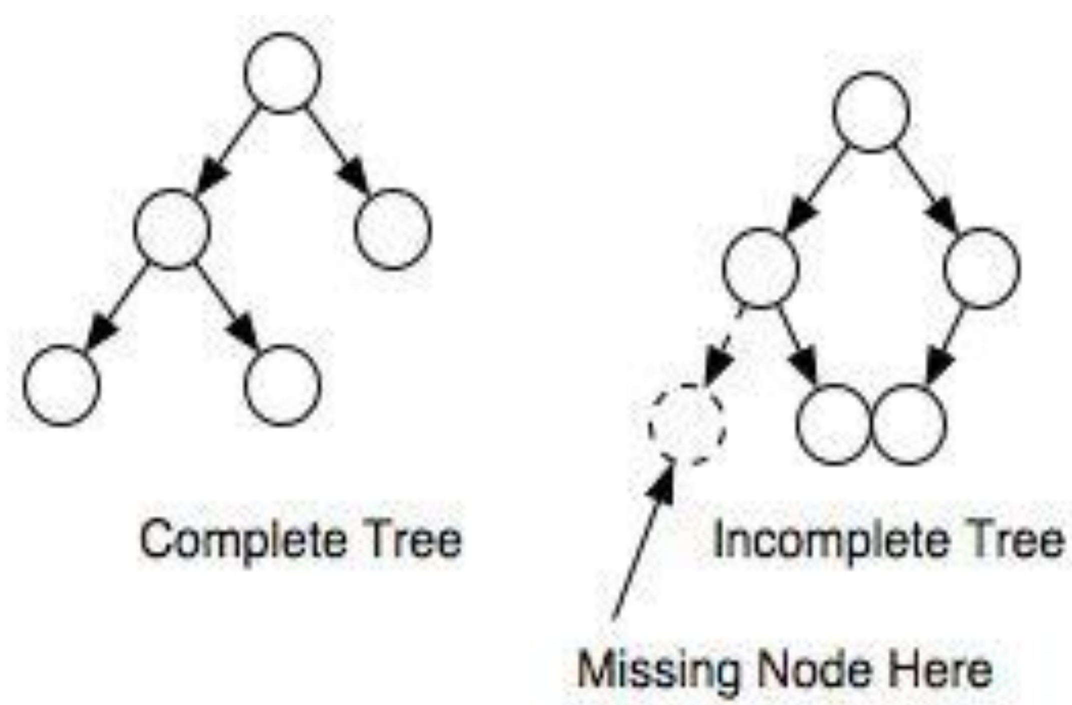
- property applies to all nodes in tree (i.e. root contains largest value in that subtree)



Complete Tree Property

Heaps are "complete trees“

- every level is filled in before adding a node to the next level
- the nodes in a given level are filled in from left to right, with no breaks.



Heap Implementations

BSTs are typically implemented as linked data structures

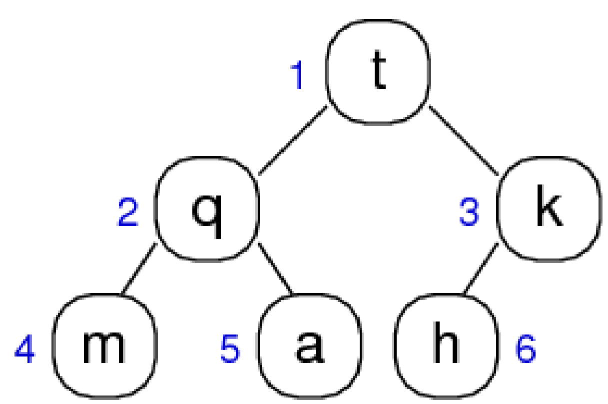
Heaps CAN be implemented as linked data structures

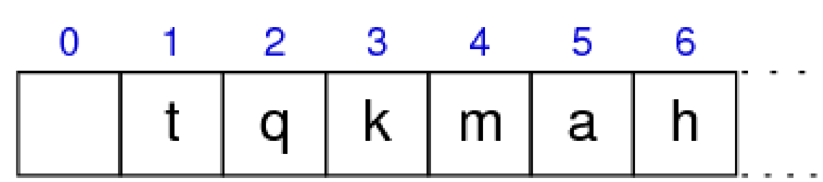
- Heaps are TYPICALLY implemented via arrays.
- The property of being **complete** makes array implementations suitable

Array based heap implementation

Simple index calculations allow navigation through the tree:

- left child of node at index i is located at 2i
- right child of node at index i is located at 2i+1
- parent of node at index i is located at i/2





Array based heap implementation

Heap data structure:

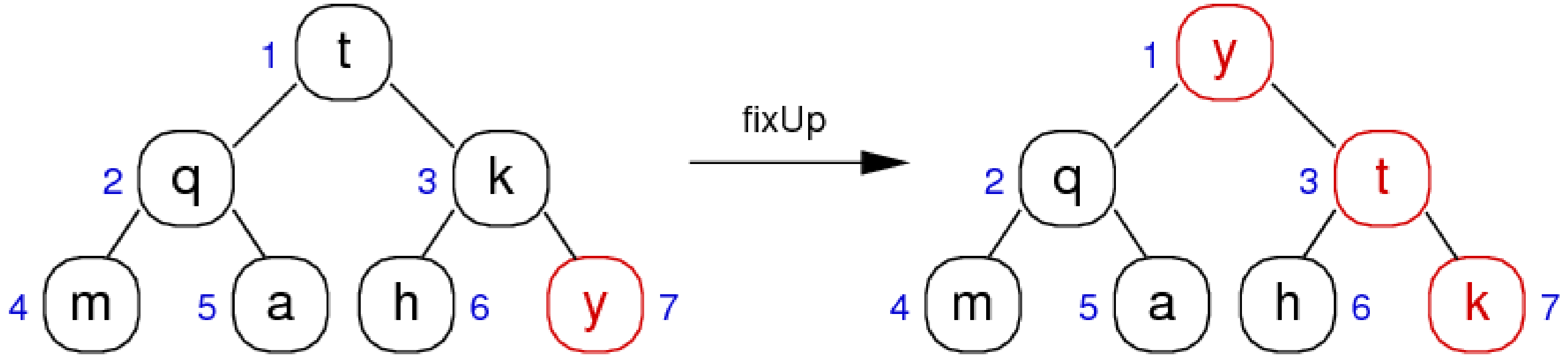
```
typedef struct HeapImp {
        Item *items; // array of Items
        int nitems;   // #items in array
}
HeapImp;

typedef HeapImp *Heap;
```

HEAP INSERTION : BOTTOM-UP HEAPIFY

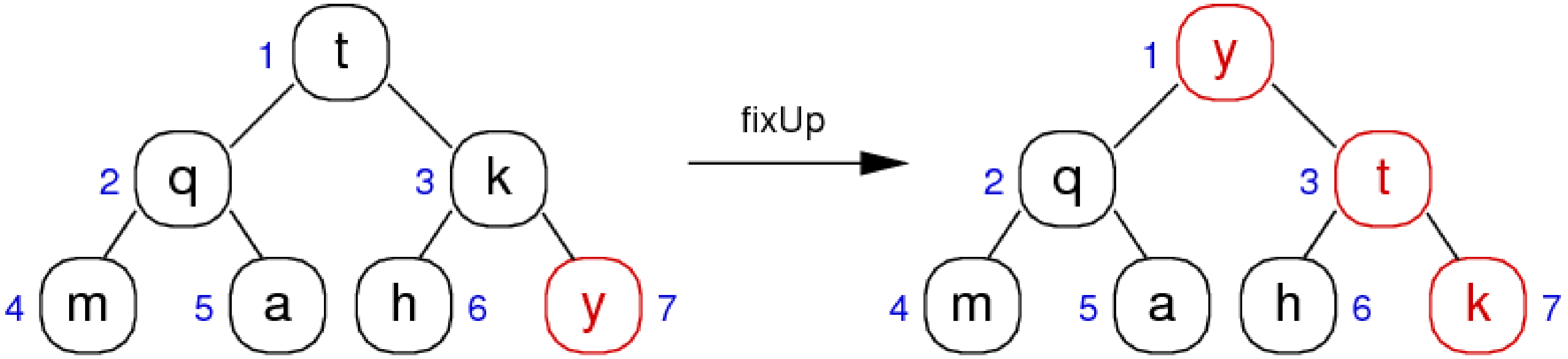
Insertion is a two-step process

1. add new element at bottom-most, rightmost position
2. reorganise values along path to root to restore heap property



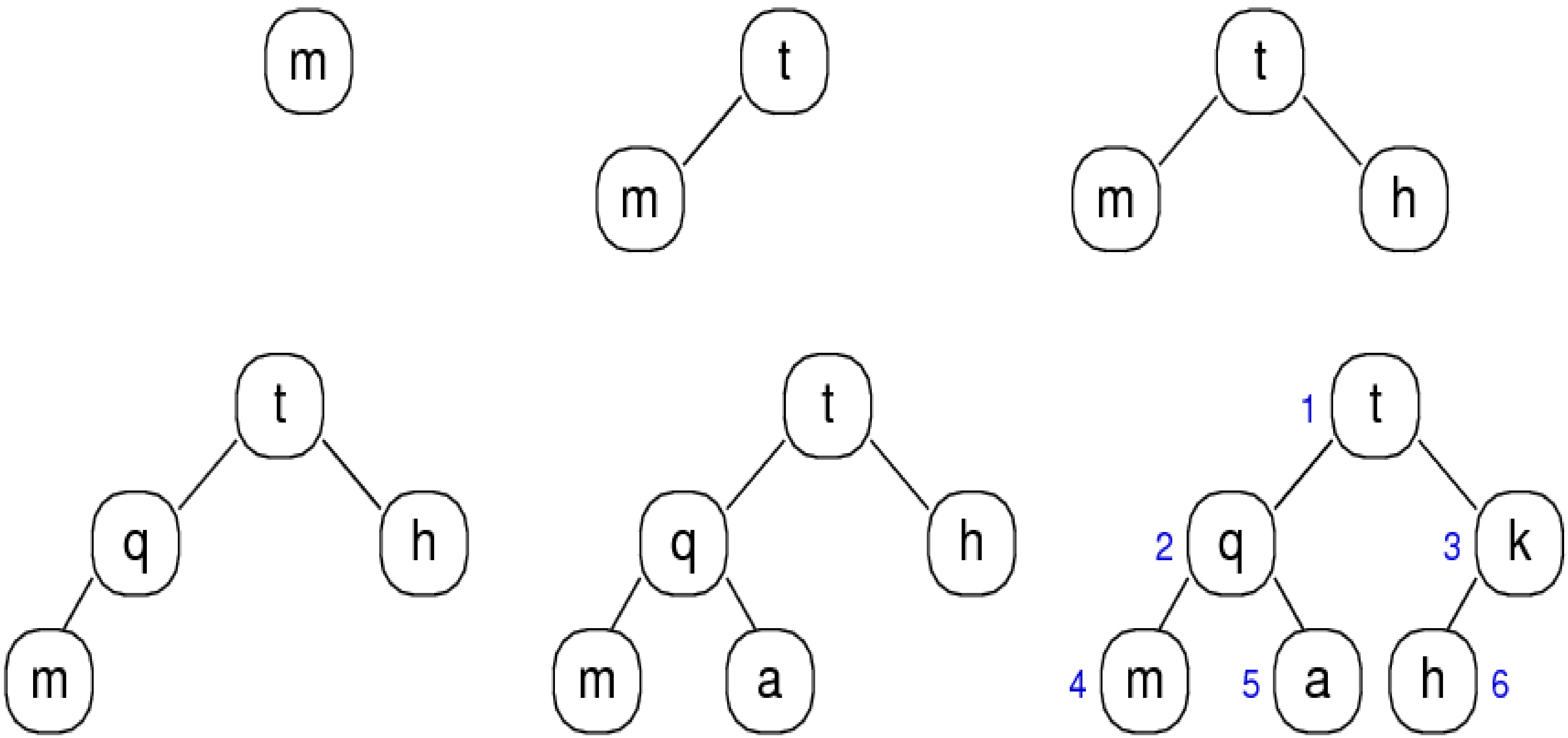
HEAP INSERTION FIX-UP CODE

```
// force value at a[k] into correct position
void fixUp(Item a[], int k) {
    while (k > 1 && less(a[k/2],a[k])) {
        swap(a, k, k/2);
        k = k/2; // integer division
    }
}
```



HEAP INSERTION

Items inserted in order m t h q a k

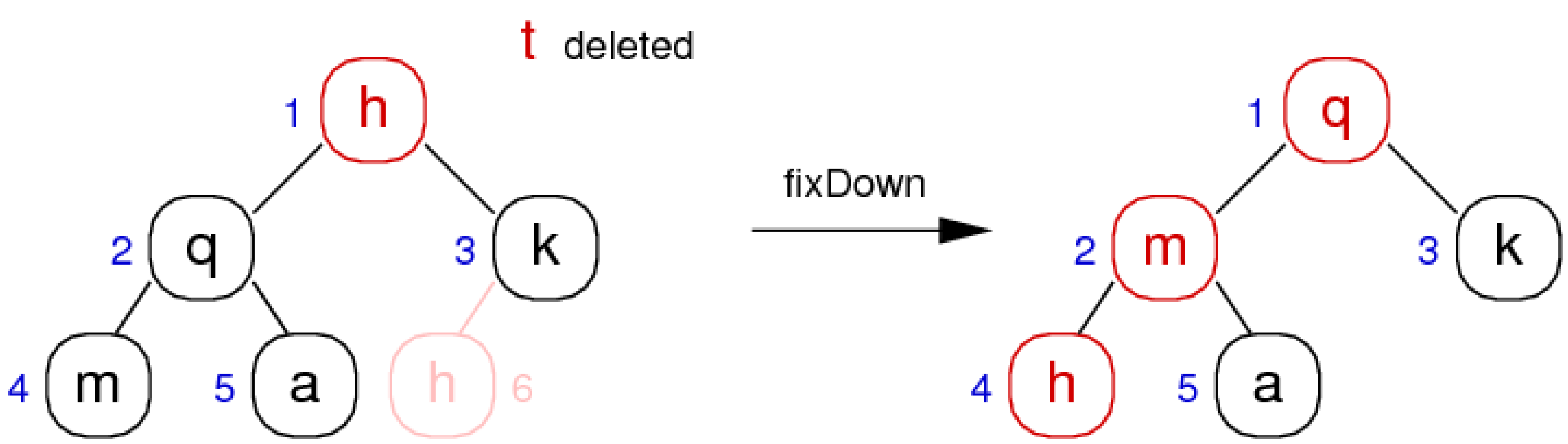


Deletion with Heaps – Top-Down Heapify

Deletion is a three-step process

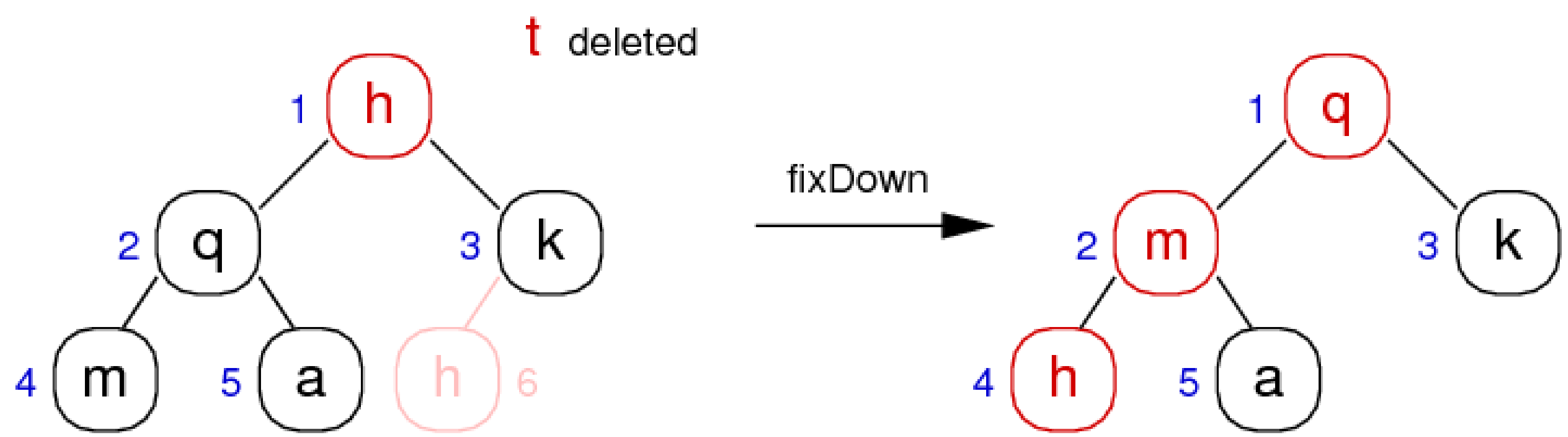
To delete node at position, k

1. replace node[k] by bottom-most, rightmost value
2. remove bottom-most, rightmost value
3. restore heap by reorganizing values by moving down the heap, exchanging node[k] with the larger of the node's children, stopping when node[k] is not smaller than either child or bottom is reached.



HEAP DELETION FIX-DOWN CODE

```
void fixDown(Item a[], int k) {
    int done = 0;
    while (2*k <= N && !done) {
        int j = 2*k; //choose larger of two children
        if (j < N && less(a[j], a[j+1])){
            j++;
        }
        if (!less(a[k], a[j])){
            done =1;
        }else{
            swap(a, k, j);
            k = j;
        }
    }
}
```



Exercise:

Show the construction of the max heap produced by inserting

H E A P S F U N

Show the heap after an item is deleted.

Show the heap after another item is deleted.

Heaps as Priority Queues

Heaps are typically used for implementing Priority Queues

- priorities determined by order on keys
- new items added initially at lower-most, right-most leaf
- then new item "drifts up" to appropriate level in tree
- items are always deleted by removing root (top priority)

Since heaps are *dense* trees, depth = $floor(log_2 N)+1$

Insertion cost = $O(logN)$,   Deletion cost = $O(logN)$