COMP3421

Modeling, Bezier Curves, L-Systems, VBOs
Curves

We want a general purpose solution for drawing *curved lines and surfaces*. It should:

- Be easy and intuitive to draw curves
- Support a wide variety of shapes, including both standard circles, ellipses, etc and "freehand" curves.
- Be computationally cheap.
Parametric curves

It is generally useful to express curves in parametric form:

\[
\begin{pmatrix}
p_1 \\
p_2 \\
p_3
\end{pmatrix} = P(t), \text{ for } t \in [0, 1]
\]

Eg:

\[
\begin{pmatrix}
x \\
y
\end{pmatrix} = \begin{pmatrix}
\cos 2\pi t \\
\sin 2\pi t
\end{pmatrix}
\]
Interpolation

Trigonometric operations like \( \sin() \) and \( \cos() \) are **expensive** to calculate.

We would like a solution that involves **fewer** floating point operations.

We also want a solution which allows for **intuitive curve design**.

Interpolating control points is a good solution to both these problems.
Linear interpolation

$$P(t) = (1 - t)P_0 + tP_1$$

Good for straight lines.
Linear function: Degree 1
2 control points: Order 2
Quadratic interpolation

\[ P(t) = (1 - t)^2 P_0 + 2t(1 - t) P_1 + t^2 P_2 \]

Interpolates (passes through) \( P_0 \) and \( P_2 \).
Approximates (passes near) \( P_1 \).
Tangents at \( P_0 \) and \( P_2 \) point to \( P_1 \).
Curves are all parabolas.
de Casteljau Algorithm

The quadratic interpolation above can be computed as three linear interpolation steps:
de Casteljau Algorithm

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\[ P_{01}(t) = (1 - t)P_0 + tP_1 \]
de Casteljau Algorithm

The quadratic interpolation above can be computed as three linear interpolation steps:

\[ P_{12}(t) = (1 - t)P_1 + tP_2 \]
de Casteljau Algorithm

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\[ P(t) = (1 - t)P_{01} + tP_{12} \]
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The quadratic interpolation above can be computed as three linear interpolation steps:

\[
P(t) = (1 - t)P_{01} + tP_{12}
\]

\[
P(t) = (1 - t)^2P_0 + 2t(1 - t)P_1 + t^2P_2
\]
de Casteljau Algorithm

\[ P_{01}(t) = (1-t)P_0 + tP_1 \]

\[ P_{12}(t) = (1-t)P_1 + tP_2 \]

\[ P(t) = (1-t)P_{01} + tP_{12} \]

\[ = (1-t) ((1-t)P_0 + tP_1) + t((1-t)P_1 + tP_2)) \]

\[ = (1-t)^2P_0 + 2t(1-t)P_1 + t^2P_2 \]
Exercise

Using de Casteljau’s algorithm calculate the point at \( t = 0.75 \) for the quadratic Bezier with the following control points.

\((0,0) \ (4,8) \ (12,4)\)

Confirm your answer using the equation

\[
P(t) = (1 - t)^2 P_0 + 2t(1 - t) P_1 + t^2 P_2
\]
Exercise Solution

\[ P_{01}(0.75) = (0.25)(0,0) + 0.75(4,8) = (3,6) \]

\[ P_{12}(0.75) = (0.25)(4,8) + 0.75(12,4) \]

\[ = (1,2) + (9,3) = (10,5) \]

\[ P_{012}(0.75) = (0.25)P_{01} + 0.75P_{12} \]

\[ = (0.25)(3,6) + 0.75(10,5) \]

\[ = (0.75, 1.25) + (7.5, 3.75) \]

\[ = (8.25, 5.25) \]
Exercise Solution

Or by using the final formula instead:

\[ P(0.75) = (1-t)^2P_0 + 2t(1-t)P_1 + t^2P_2 \]

\[ = 0.25^2(0,0) + \]

\[ 2 \times 0.75 \times 0.25 (4,8) + \]

\[ 0.75^2 (12,4) \]

\[ = (8.25, 5.25) \]
Cubic interpolation

\[ P(t) = (1 - t)^3 P_0 + 3t(1 - t)^2 P_1 + 3t^2(1 - t) P_2 + t^3 P_3 \]

Interpolates (passes through) \( P_0 \) and \( P_3 \).
Approximates (passes near) \( P_1 \) and \( P_2 \).
Tangents at \( P_0 \) to \( P_1 \) and \( P_3 \) to \( P_2 \).
A variety of curves.
de Casteljau

\[ P_0 \rightarrow P_1 \rightarrow P_2 \rightarrow P_3 \]
de Casteljau
de Casteljau

P₀₁

P₀₁₂

P₁₂

P₁₂₃

P₂₃

P₂

P₃
de Casteljau

$P_0$, $P_1$, $P_2$, $P_3$, $P_{012}$, $P_{123}$

$t=0.5$
de Casteljau
Degree and Order

**Linear Interpolation:** Degree one curve \((m=1)\), Second Order (2 control points)

**Quadratic Interpolation:** Degree two curve \((m=2)\), Third Order (3 control points)

**Cubic Interpolation:** Degree three curve \((m=3)\), Fourth Order (4 control points)

**Quartic Interpolation:** Degree four curve \((m=4)\), Fifth Order (5 control points)

Etc…
Bézier curves

This family of curves are known as Bézier curves.

They have the general form:

\[ P(t) = \sum_{k=0}^{m} B^m_k(t) P_k \]

where \( m \) is the degree of the curve and \( P_0 \ldots P_m \) are the control points.
Bernstein polynomials

The coefficient functions $B^m_k(t)$ are called Bernstein polynomials. They have the general form:

$$B^m_k(t) = \binom{m}{k} t^k (1 - t)^{m-k}$$

where:

$$\binom{m}{k} = \frac{m!}{k!(m-k)!}$$

is the binomial function.
Binomial Function

Remember Pascal’s triangle

```
                  1
                 1 1
                1 2 1
               1 3 3 1
              1 4 6 4 1
             1 5 10 10 5 1
```
Bernstein polynomials

\[ B_k^m(t) = \binom{m}{k} t^k (1 - t)^{m-k} \]

For the most common case, \( m = 3 \):

\[
\begin{align*}
B_0^3(t) &= (1 - t)^3 \\
B_1^3(t) &= 3t(1 - t)^2 \\
B_2^3(t) &= 3t^2(1 - t) \\
B_3^3(t) &= t^3
\end{align*}
\]

\[ P(t) = (1 - t)^3 P_0 + 3t(1 - t)^2 P_1 + 3t^2(1 - t) P_2 + t^3 P_3 \]
Bernstein Polynomials for $m = 3$
Exercise

\[ B_k^m(t) = \binom{m}{k} t^k (1 - t)^{m-k} \]

What are the Bernstein polynomials for \( m = 4 \)?
Solution

\[ B_k^m(t) = \binom{m}{k} t^k (1 - t)^{m-k} \]

What are the Bernstein polynomials for \( m = 4 \)?

\[ B_0^4(t) = (1 - t)^4 \]
\[ B_1^4(t) = 4t(1 - t)^3 \]
\[ B_2^4(t) = 6t^2(1 - t)^2 \]
\[ B_3^4(t) = 4t^3(1 - t) \]
\[ B_3^4(t) = t^4 \]
Properties

Bézier curves \textit{interpolate} their endpoints and \textit{approximate} all intermediate points.

Bézier curves are \textit{convex combinations} of points:

\[
\sum_{k=0}^{m} B_k^m(t) = 1
\]

Therefore they are \textit{invariant} under affine transformation. The transformation of a Bézier curve is the curve based on the transformed control points.
Properties

A Bézier curve lies within the **convex hull** of its control points:
Tangents

The tangent vector to the curve at parameter $t$ is given by:

$$\frac{dP(t)}{dt} = \sum_{k=0}^{m} \frac{dB_k^m(t)}{dt} P_k$$

$$= m \sum_{k=0}^{m-1} B_k^{m-1}(t)(P_{k+1} - P_k)$$

This is a Bézier curve of degree $(m-1)$ on the vectors between control points.
Exercise

Compute the tangent to at \( t = 0.25 \) for a quadratic Bezier curve with control points \((0,0)\ (4,4)\ (8,2)\)

\[
P'(t) = 2 \times \left[(1-t)(P_1-P_0) + t(P_2-P_1)\right]
\]

\[
P'(0.25) = 2 \times \left[ (0.75)((4,4)-(0,0)) + 0.25((8,2)-(4,4)) \right]
\]

\[
= 2 \times \left[ (0.75)(4,4) + 0.25(4,-2) \right]
\]

\[
= 2 \times \left[ (3,3) + (1, -0.5) \right] = (8,5)
\]
Problem: Polynomial Degree

The degree of the Bernstein polynomials used is coupled to the number of control points: $L+1$ control points is a combination of $L$-degree polynomials.

High degree polynomials are expensive to compute and are vulnerable to numerical rounding errors.
Problem: Local control

These curves suffer from non-local control.

Moving one control point affects the entire curve.

Each Bernstein polynomial is active (non-zero) over the entire interval \((0,1)\). The curve is a blend of these functions so every control point has an effect on the curve for all \(t\) from \((0,1)\)
Splines

A spline is a smooth piecewise-polynomial function (for some measurement of smoothness).

The places where the polynomials join are called knots.

A joined sequence of Bézier curves is an example of a spline.
Local control

A spline provides local control.

A control point only affects the curve within a limited neighbourhood.
Bézier splines

We can draw longer curves as sequences of Bézier sections with common endpoints:
3D Modeling

What if we are sick of teapots?

How can we make our own 3d meshes that are not just cubes?

We will look at simple examples along with some clever techniques such as

• Extrusion
• Revolution
Exercise: Cone

How can we model a cone?

There are many ways.

Simple way: Make a circle using a triangle fan parallel to the x-y plane. For example at $z = -3$

Change to middle point to lie at a different z-point for example $z = -1$. 
Extruding shapes

Extruded shapes are created by sweeping a 2D polygon along a line or curve.

The simplest example is a prism.
Variations

One copy of the prism can be translated, rotated or scaled from the other.
Segmented Extrusions

A square $P$ extruded three times, in different directions with different tapers and twists. The first segment has end polygons $M_0 P$ and $M_1 P$, where the initial matrix $M_0$ positions and orients the starting end of the tube. The second segment has end polygons $M_1 P$ and $M_2 P$, etc.
Segmented extrusions

We can extrude a polygon along a path by specifying it as a series of transformations.

\[
poly = P_0, P_1, \ldots, P_k
\]
\[
path = M_0, M_1, \ldots, M_n
\]

At each point in the path we calculate a cross-section:

\[
poly_i = M_i P_0, M_i P_1, \ldots, M_i P_k
\]
Segmented Extrusion

Sample points along the spine using different values of t

For each t:

• generate the current point on the spine
• generate a transformation matrix
• multiply each point on the cross section by the matrix.
• join these points to the next set of points using quads/triangles.
Segmented Extrusion Example

For example we may wish to extrude a circle cross-section around a helix spine.

helix $C(t) = (\cos(t), \sin(t), bt)$.
Transformation Matrix

How can we automatically generate a matrix to transform our cross-section by?

We need the origin of the matrix to be the new point on the spine. This will translate our cross-section to the correct location.

Which way will our cross-section be oriented? What should i, j and k of our matrix be?
We can get the curve values at various points $t_i$ and then build a polygon perpendicular to the curve at $C(t_i)$ using a Frenet frame.
Example

a). Tangents to the helix.  b). Frenet frame at various values of $t$, for the helix.
Frenet Frame

Once we calculate the tangent to the spine at the current point, we can use this to calculate normals.

We then use the tangent and the 2 normals as i, j and k vectors of a co-ordinate frame.

We can then build a matrix from these vectors, using the current point as the origin of the matrix.
Frenet frame

We align the \( \mathbf{k} \) axis with the (normalised) tangent, and choose values of \( \mathbf{i} \) and \( \mathbf{j} \) to be perpendicular.

\[
\begin{align*}
\phi &= \Phi(t) \\
\mathbf{k} &= \hat{\Phi}'(t) \\
\mathbf{i} &= \begin{pmatrix} -k_2 \\ k_1 \\ 0 \end{pmatrix} \\
\mathbf{j} &= \mathbf{k} \times \mathbf{i}
\end{align*}
\]
Frenet Frame Calculation

Finding the tangent (our k vector):

1. Using maths. Eg for

   \[ C(t) = (\cos(t), \sin(t), bt) \]
   \[ T(t) = \text{normalise}(-\sin(t), \cos(t), b) \]

2. Or just approximate the tangent

   \[ T(t) = \text{normalise}(C(t+1) - C(t-1)) \]
Frenet Frame Calculation

If our tangent at \( t \) is the vector

\[
T(x,y,z)
\]

We can use the normal

\[
N(-y,x,0). \text{ This will be our } i \text{ vector}
\]

To find the other normal we simply do \( k \times i \)
Revolution

A surface with radial symmetry (i.e. a round object, like a ring, a vase, a glass) can be made by sweeping a half cross-section around an axis.
Revolution

Take your 2d function which can generate points for $X(t)$ and $Y(t)$ and sample them for different values of $t$ and angles of $a$ (angle of rotation around axis).

//Revolution around the Y-axis
$P(t,a) = (X(t) \cos a, Y(t), X(t) \sin a)$
A Lindenmayer System (or L-System) is a method for producing fractal structures. They were initially developed as a tool for modelling plant growth.

nmayer-power.html
Rewrite rules

An L-system is a formal grammar: a set of symbols and rewrite rules. Eg:

Symbols:
A, B, +, -

Rules:
A → B - A - B
B → A + B + A
Iteration

We start with a given string of symbols and then iterate, replacing each on the left of a rewrite rule with the string on the right.

A

B - A - B
A + B + A - B - A - B - A + B + A
B - A - B + A + B + A + B - A - B - ...

...
Drawing

Each string has a **graphical interpretation**, usually using turtle graphics commands:

A = draw forward 1 step

B = draw forward 1 step

+ = turn left 60 degrees

- = turn right 60 degrees
Sierpinski Triangle

This L-System generates the fractal known as the Sierpinski Triangle:
Parameters

We can add parameters to our rewrite rules to handle variables like scaling:

A(s) → B(s/2) - A(s/2) - B(s/2)

B(s) → A(s/2) + B(s/2) + A(s/2)

A(s) : draw forward s units
B(s) : draw forward s units
Push and Pop

We can also use a LIFO stack to save and restore global state like position and heading:

A → B [ + A ] - A
B → B B

A : forward 10      B : forward 10
+: rotate 45 left    - : rotate 45 right
[ : push            ] : pop ;
Stochastic

We can add multiple productions with weights to allow random selection:

(0.5) $A \rightarrow B \ [ A ] \ A$

(0.5) $A \rightarrow A$

$B \rightarrow B \ B$
Example

(0.5) X → F - [ [ X ] + X ] + F [ + F X ] - X
(0.5) X → F - F [ + F X ] + [ [ X ] + X ] - X
F → F F
3D L-Systems

We can build 3D L-Systems by allowing symbols to translate to models and transformations of the coordinate frame.

C : draw cylinder mesh
   F : translate(0,0,10)
   X : rotate(10, 1, 0, 0)
   Y : rotate(10, 0, 1, 0)
   S : scale(0.5, 0.5, 0.5)
Example

A -> A - A + A - A : + rotate 45 (CW)
B -> BA : - rotate -90
After 1 iteration? : [ push
After 2 iterations? : ] pop
After 3 iterations?
Example in Format
For Web Demo

-> S
1 A [ + B ] + A
-> A
1 A - A + A - A
-> B
1 BA

: A
forward 10
: +
rotate 45
: -
rotate -90
: [
push
: ]
pop
Example Generation

S -> A [ + B ] + A
A -> A - A + A - A
B -> BA

After 1 iteration?
A [ + B ] + A

After 2 iterations?

After 3 iterations?
A - A + A - A - A - A + A
-A + A - A + A - A ETC
Example Drawing

After 1 iteration?
A [ + B ] + A

: A forward 10
: + rotate 45 (CW)
: - rotate -90
: [ push
: ] pop
Example Drawing

After 2 iterations?


: A forward 10
: + rotate 45 (CW)
: - rotate -90
: [ push
: ] pop
3 iterations?

Algorithmic Botany

You can read a LOT more here:

http://algorithmicbotany.org/papers/
Immediate Mode

Primitives are sent to pipeline and displayed right away

More calls to OpenGL commands

No memory of graphical entities on server side
- Primitive data lost after drawing which is inefficient if we want to draw object again
Immediate Mode Example

```c
glBegin(GL2.GL_TRIANGLES);
  gl.glVertex3d(0,2,-4);
  gl.glVertex3d(-2,-2,-4);
  gl.glVertex3d(2,-2,-4);
}gl.glEnd();
```
Retained Mode

Store data in the graphics card’s memory instead of retransmitting every time

OpenGL can store data in *Vertex Buffer Objects* on Graphics Card
Vertices

As we know a vertex is a collection of attributes:

- position
- colors
- normal
- etc

**VBOs** store all this data for all the primitives you want to draw at any one time.

**VBOs** store this data on the server/graphics card
Client Side Data

// Suppose we have 6 vertices with
// positions and corresponding colors in
// our jogl program

float positions[] = {0,1,-1, -1,-1,-1, 
                    1,-1,-1, 0, 2,-4, 
                    -2,-2,-4, 2,-2,-4};

float colors[] = {1,0,0, 0,1,0, 
                  1,1,1, 0,0,0, 
                  0,0,1, 1,1,0};
Client Side Data

In jogl the VBO commands do not take in arrays.

We need to put them into containers which happen to be called **Buffers**. These are still client side containers and not on the graphics card memory.

```java
FloatBuffer posData = Buffers.newDirectFloatBuffer(positions);
FloatBuffer colorData = Buffers.newDirectFloatBuffer(cols);
```

Our data is now ready to be loaded into a VBO.
Vertex Buffer Objects

VBOs are allocated by `glGenBuffers` which creates int IDs for each buffer created.

```c
//For example generating two buffers
int bufferIDs[] = new int[2];
gl.glGenBuffers(2, bufferIDs, 0);
```
VBO Targets

There are different types of buffer objects.

For example:

   GL_ARRAY_BUFFER is the type used for storing vertex attribute data

   GL_ELEMENT_ARRAY_BUFFER can be used to store indexes to vertex attribute array data
Indexing

Without indexing

With indexing

With indexing you need an extra VBO to store index data.
Binding VBO targets

//Bind GL_ARRAY_BUFFER to the VBO. This makes the buffer the current buffer for reading/writing vertex array data

gl.glBindBuffer(GL2.GL_ARRAY_BUFFER, bufferIDs[0]);
Vertex Display Buffers

// Upload data into the current VBO
gl.glBufferData(int target,
               int size,
               Buffer data,
               int usage);

//target – GL2.GL_ARRAYBUFFER,
//GL2.GL_ELEMENT_ARRAY_BUFFER etc
//size – of data in bytes
//data – the actual data
//usage – a usage hint
GL2.GL_STATIC_DRAW: data is expected to be used many times without modification. Optimal to store on graphics card.

GL2.GL_STREAM_DRAW: data used only a few times. Not so important to store on graphics card.

GL2.GL_DYNAMIC_DRAW: data will be changed many times
// Upload data into the current VBO
// For our example if we were only
// loading positions we could use

gl.glBufferData(GL2.GL.ARRAYBUFFER,
    posData.length*Float.BYTES,
    posData,
    GL2.GL_STATIC_DRAW);
Vertex Display Buffers

// Upload data into the current VBO
// For our example if we were wanting
// to load position and color data
// we could create an empty buffer of the
// desired size and then load in each
// section of data using glBufferSubData

gl.glBufferData(GL2.GL_ARRAY_BUFFER,
positions.length*Float.BYTES +
colors.length*Float.BYTES,
null, GL2.GL_STATIC_DRAW);
Vertex Display Buffers

//Specify part of data stored in the current VBO once buffer has been made
//For example vertex positions and color data may be stored back to back

gl.glBufferSubData(int target, //in bytes
                   int offset, //in bytes
                   int size,   //in bytes
                   Buffer data
);
Vertex Display Buffers

// Specify part of data stored in the current VBO once buffer has been made
// For example vertex positions and color data may be stored back to back

```
gl.glBufferSubData(GL2.GL_ARRAY_BUFFER, 0, positions.length*Float.BYTES, posData);

gl.glBufferSubData(GL2.GL_ARRAY_BUFFER, positions.length*Float.BYTES, // offset colors.length*Float.BYTES, colorData);
```
VBOs

Application Program

- Vertex Data []
- Color Data []

Graphics Card

- VBO ID
- GL_ARRAY_BUFFER
- VBO
  - Vertex Data []
  - Color Data []
Using VBOs

All we have done so far is copy data from the client program to the Graphics card. This is done when `glBufferData` or `glBufferSubData` is called.

We need to tell the graphics pipeline what is in the buffer – for example which parts of the buffer have the position data vs the color data.
Using VBOs

To tell the graphics pipeline that we want it to use our vertex position and color data

```cpp
//Enable client state
gl.glEnableClientState(GL2.GL_VERTEX_ARRAY);
gl.glEnableClientState(GL2.GL_COLOR_ARRAY);

//For other types of data
```
Using VBOs with Shaders

To link your vbo to your shader inputs (you get to decide what they are called and used for), instead of `gl.glEnableClientState`,

```c
//assuming the vertex shader has
//in vec4 vertexPos;

int vPos =
   gl.glGetAttribLocation(shaderprogram, "vertexPos");

gl.glEnableVertexAttribArray(vPos);
```
Using VBOs

//Tell OpenGL where to find data
gl.glVertexPointer(int size,
    int type,
    int stride,
    long vboOffset);

//size – number of co-ords per vertex
//type – GL2.GL_FLOAT etc
//stride – distance in bytes between beginning of vertex locations.
//vboOffset – offset in number of bytes of data location
Using VBOs

//Tell OpenGL where to find other data.
//Must have 1-1 mapping with the vertex array

gl.glColorPointer(int size,
                 int type,
                 int stride,
                 long vboOffset);

gl.glNormalPointer(int type,
                    int stride,
                    long vboOffset);
Using VBOs

// Tell OpenGL where to find data
// In our example each position has 3
// float co-ordinates. Positions are not
// interleaved with other data and are
// at the start of the buffer
gl.glVertexPointer(3, GL.GL_FLOAT, 0, 0);

// In our example color data is found
// after all the position data
gl.glColorPointer(3, GL.GL_FLOAT, 0, positions.length*Float.BYTES);
Using VBOs with Shaders

//Tell OpenGL where to find data

```java
gl.glVertexAttribPointer(int index,
    int size, int type, boolean normalised,
    int stride, long vboOffset);

//index - shader attribute index
//normalised - whether to normalize the data
```

gl.glVertexAttribPointer(vPos, 3, GL.GL_FLOAT, false, 0, 0);
VBOs

Application Program

- Vertex Data []
- Color Data []

Graphics Card

- VBO ID
- GL_ARRAY_BUFFER
- VBO
- GL_VERTEX_ARRAY
- GL_COLOR_ARRAY
- Vertex Data []
- Color Data []
Drawing with VBOs

// Draw something using the data sequentially

gl.glDrawArrays(int mode,
                int first,
                int count);

//mode – GL_TRIANGLES etc
//first – index of first vertex to be drawn
//count – number of vertices to be used.
Drawing with VBOs

//In our example we have data for 2 triangles, so 6 vertices
//and we are starting at the vertex at index 0
gl.glDrawArrays(GL2.GL_TRIANGLES, 0, 6);

//This would just draw the second triangle
gl.glDrawArrays(GL2.GL_TRIANGLES, 3, 6);
Indexed Drawing

// Draw something using indexed data
gl.glDrawElements(int mode, int count,
                int type, long offset);

// mode - GL_TRIANGLES etc
// count - number of indices to be used.
// type - type of index array - should be
// unsigned and smallest type possible.
// offset - in bytes!
//Suppose we want to use indexes to
//access our data
short indexes[] = {0,1,5,3,4,2};
ShortBuffer indexedData = Buffers.newDirectShortBuffer(indexes);

//Set up another buffer for //the
indexes
gl.glBindBuffer(GL2.GL_ELEMENT_ARRAY_BUFFER, bufferIDs[1]);
Indexed Drawing

//load index data
gl.glBufferData(
    GL2.GL_ELEMENT_ARRAY_BUFFER,
    indexes.length * Short.BYTES,
    indexData,
    GL2.GL_STATIC_DRAW);

//draw the data
gl.glDrawElements(GL2.GL_TRIANGLES, 6,
    GL2.GL_UNSIGNED_SHORT, 0);
Updating a VBO

- Copy new data into the bound VBO with `gl.glBufferData()` or `glBufferSubData()`.
- Map the buffer object into client's memory, so that client can update data with the pointer to the mapped buffer `glMapBuffer()`.
Drawing Multiple Objects

Must make many calls each time we draw each new shape.

glBindBuffer
glVertexPointer
glColorPointer
glNormalPointer
etc
Vertex Array Object (VAO)

Encapsulates vbo and vertex attribute states to rapidly switch between states using one OpenGL call.

```java
gl.glBindVertexArray(vaoIDs[0]);
```

First generate vao ids.

```java
int vaoIDs[] = new int[2];
gl.glGenVertexArrays(2, vaoIDs, 0);
```
Set up VAOs

//Assume vbos and vao ids have been set up.
gl.glBindVertexArray(vaoIds[0]);
gl.glBindBuffer(GL2.GL_ARRAY_BUFFER,vboIds[0]);
gl.glEnableClientState...
gl.glVertexPointer... //etc other calls

//Assume vbos and vao ids have been set up.
gl.glBindVertexArray(vaoIds[1]);
gl.glBindBuffer(GL2.GL_ARRAY_BUFFER,vboIds[1]);
gl.glEnableClientState..
gl.glVertexPointer
//etc other calls
VAO switching

// Once vaos have been set up
gl.glBindVertexArray(vaoIds[0]);
gl.glDrawArrays(GL2.GL_TRIANGLES, 0, N);
gl.glBindVertexArray(vaoIds[1]);
gl.glDrawArrays(GL2.GL_TRIANGLES, 0, M);
// Use no vao
gl.glBindVertexArray(0);
Deleting a VBOs and VAOs

To free VBOs and VAOs once you do not need them anymore.

```cpp
gl.glDeleteBuffers(2, vboIds, 0);
gl.glDeleteVertexArray(2, vaoIds, 0);
```