Particle systems

Some visual phenomena are best modelled as collections of small particles.

Examples: rain, snow, fire, smoke, dust
Particle systems

Particles are usually represented as small textured quads or point sprites – single vertices with an image attached.

They are billboarded, i.e transformed so that they are always face towards the camera.
Billboarding

Camera

- camera viewing direction
- plane perpendicular to the camera
Billboarding

An approximate form of billboarding can be achieved by having polygons face a plane perpendicular to the camera
Billboarding

We can apply this approximation by altering the model-view matrix.

\[
\begin{pmatrix}
  i_x & j_x & k_x & \phi_x \\
  i_y & j_y & k_y & \phi_y \\
  i_z & j_z & k_z & \phi_z \\
  0   & 0   & 0   & 1 \\
\end{pmatrix}
\Rightarrow
\begin{pmatrix}
  |i| & 0   & 0   & \phi_x \\
  0   & |j| & 0   & \phi_y \\
  0   & 0   & |k| & \phi_z \\
  0   & 0   & 0   & 1 \\
\end{pmatrix}
\]
float modelview[16];

gl.glPushMatrix();

// get the current modelview matrix
gl.glGetFloatv(GL_MODELVIEW_MATRIX, modelview);

// modify the matrix
billboard(modelview);

gl.glLoadMatrixf(modelview);

drawObject(gl);

gl.glPopMatrix();
Particle systems

Particles are created by an emitter object and evolve over time, usually changing position, size, colour.
Particle evolution

Usually the rules for particle evolution are simple local equations:

- interpolate from one colour to another over time
- move with constant speed or acceleration.

To simulate many particles it is important these update steps are kept simple and fast.
Particles on the GPU

Particle systems are well suited to implementation as vertex shaders.

The particles can be represented as individual point vertices.

A vertex shader can compute the position of each particle at each moment in time.
Particle System

uniform vec3 vel;
uniform float g, t;

void main(){
    vec3 pos;
    pos.x = gl_Vertex.x + vel.x*t;
    pos.y = gl_Vertex.y + vel.y*t + g*t*t;
    pos.z = gl_Vertex.z + vel.z*t;

    gl_Position =
        ModelViewProjectionMatrix*vec4(pos,1);
}
Exercise

Adapt the fireworks example to create a tornado.
Solution

See code.
The graphics pipeline

Model

Model-View Transform

Model Transform → View Transform

Illumination

Projection transformation

Clipping

Perspective division

Viewport

Rasterisation

Texturing

Hidden surface removal

Frame buffer

Display

User
Rasterisation

Rasterisation is the process of converting lines and polygons represented by their vertices into fragments.

Fragments are like pixels but include color, depth, texture coordinate. They may also never make it to the screen due to hidden surface removal or culling.
Rasterisation

This operation needs to be **accurate** and **efficient**.

For this reason we prefer to use simple integer calculations.

All are calculations are now in 2D screen space.
Drawing lines

\[(x, y)\]
\[(x_0, y_0)\]
\[(x_1, y_1)\]

\[
m = \frac{y_1 - y_0}{x_1 - x_0}
\]

\[
b = y_0 - mx_0
\]

\[
y = mx + b
\]
double m = (y1-y0)/(double)(x1-x0);

double b = y0 - m * x0;

for (int x = x0; x <= x1; x++) {
    int y = round(m * x + b);
    drawPixel(x, y);
}

Drawing lines - bad
Problems

• Floating point math is slow and creates rounding errors
  • Floating point multiplication, addition and round for each pixel

• Code does not consider:
  • Points are not connected if $m > 1$
  • Divide by zero if $x_0 == x_1$ (vertical lines)
  • Doesn't work if $x_0 > x_1$
Example: $y = 2x$
Incremental – still bad

// incremental algorithm
double m = (y1-y0)/(double)(x1-x0);
double y = y0;
for (int x = x0; x <= x1; x++) {
    y += m; //one less multiplication
    drawPixel(x, round(y));
}
Bresenham's algorithm

We want to draw lines using only integer calculations and avoid multiplications.

Such an algorithm is suitable for fast implementation in hardware.

The key idea is that calculations are done incrementally, based on the values for the previous pixel.
Bresenham's algorithm

We shall assume to begin with that the line is in the first octant.

l.e. $x_1 > x_0$, $y_1 > y_0$ and $m \leq 1$
Bresenham’s Idea

For each x we work out which pixel we set next

The next pixel with the same y value
if the line passes below the midpoint between the two pixels

Or the next pixel with an increased y value
if the line passes above the midpoint between the two pixels
Bresenham's algorithm

- \( P(x_i, y_i) \)
- \( L(x_{i+1}, y_i) \)
- \( U(x_{i+1}, y_{i+1}) \)
- \( M \)
- \( M_1 \)
- \( M_2 \)
int y = y0;
for (int x = x0; x <= x1; x++) {
    setPixel(x,y);
    M = (x + 1, y + 1/2)
    if (M is below the line)
        y++
}
We’re on the line when:

\[ w = x_1 - x_0 \]
\[ h = y_1 - y_0 \]
\[ m = \frac{h}{w} \]
\[ y - y_0 = m(x - x_0) \]
\[ 0 = m(x - x_0) - (y - y_0) \]
Testing above/below

We’re above the line when:

\[ 0 < m(x - x_0) - (y - y_0) \]
\[ 0 < (h / w)(x - x_0) - (y - y_0) \]
\[ 0 < h(x - x_0) - w(y - y_0) \]
\[ 0 < 2h(x - x_0) - 2w(y - y_0) \]
Testing above/below

We call this value F

\[ F(x, y) = 2h(x - x_0) - 2w(y - y_0) \]

\( F(x, y) < 0 \Rightarrow (x, y) \) is below line

\( F(x, y) > 0 \Rightarrow (x, y) \) is above line
Midpoints

$P (x_i, y_i)$
Incrementally

\[ F(M) = 2h(x_0 + 1 - x_0) - 2w(y_0 + \frac{1}{2} - y_0) \]
\[ = 2h - w \]

\[ F(M_1) = 2h(x_0 + 2 - x_0) - 2w(y_0 + \frac{1}{2} - y_0) \]
\[ = F(M) + 2h \]

\[ F(M_2) = 2h(x_0 + 2 - x_0) - 2w(y_0 + \frac{3}{2} - y_0) \]
\[ = F(M) + 2h - 2w \]
int y = y0;
int w = x1 - x0; int h = y1 - y0;
int F = 2 * h - w;

for (int x = x0; x <= x1; x++) {
    setPixel(x,y);
    if (F < 0) F += 2*h;
    else {
        F += 2*(h-w);  y++;
    }
}
Example

\[ w = 8 \]
\[ h = 5 \]
\[ \text{int } F = 2 \times h - w; \]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>
Example

\[ w = 8 \]
\[ h = 5 \]
\[ 2n \in \mathbb{R}, h = -2w, \quad k = -6w; \]
Example

\[ (0,0) \rightarrow (8,5) \]

\[
\begin{align*}
w &= 8 \\
h &= 5 \\
2 \times (h - w) &= -6 \\
2 \times h &= 10
\end{align*}
\]

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-4</td>
</tr>
</tbody>
</table>
Example

\( w = 8 \)
\( h = 5 \)
\[
2 \times (h - w) = -6 \\
2 \times h = 10
\]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>
Example

\[ w = 8 \]
\[ h = 5 \]
\[ 2 \times (h - w) = -6 \]
\[ 2 \times h = 10 \]
Example

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<tbody>
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<td>2</td>
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<td>1</td>
<td>-4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>-6</td>
</tr>
</tbody>
</table>
Example

- \( w = 8 \)
- \( h = 5 \)
- \( 2 \times (h - w) = -6 \)
- \( 2 \times h = 10 \)

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<thead>
<tr>
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<tbody>
<tr>
<td>0</td>
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<td>2</td>
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<td>1</td>
<td>1</td>
<td>-4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>-6</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>-2</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>
Relaxing restrictions

Lines in the other quadrants can be drawn by symmetrical versions of the algorithm.

We need to be careful that drawing from P to Q and from Q to P set the same pixels.

Horizontal and vertical lines are common enough to warrant their own optimised code.
Polygon filling

Determining which pixels are inside a polygon is a matter of applying the edge-crossing test (from week 3) for each possible pixel.
Shared edges

Pixels on shared edges between polygons need to be drawn consistently regardless of the order the polygons are drawn, with no gaps.

We adopt a rule:

The edge pixels belong to the rightmost and/or upper polygon ie Do not draw rightmost or uppermost edge pixels
Scanline algorithm

Testing every pixel is very inefficient.

We only need to check where the result changes value, i.e. when we cross an edge.

We proceed row by row:

Calculate intersections incrementally.

Sort by x value.

Fill runs of pixels between intersections.
Active Edge List

We keep a list of active edges that overlap the current scanline.

Edges are added to the list as we pass the bottom vertex.

Edges are removed from the list as we pass the top vertex.

The edge intersection is updated incrementally.
Edges

For each edge in the AEL we store:

The $x$ value of its crossing with the current row (initially the bottom $x$ value)

The amount the $x$ value changes from row-to-row ($1/\text{gradient}$)

The $y$ value of the top vertex.
Edge table

The (inactive) edge table is a lookup table index on the y-value of the lower vertex of the edge. This allows for fast addition of new edges.

Horizontal edges are not added

In this list we store the initial values needed in the active edge list as well as the starting y value for the edge.
// For every scanline

for (y = minY; y <= maxY; y++){
    remove all edges that end at y

    for (Edge e : active) {
        e.x = e.x + e.inc;
    }

    add all edges that start at y - keep list sorted by x

    for (int i=0; i < active.size; i+=2){
        fillPixels(active[i].x, active[i+1].x,y);
    }
}
}
Example

Edge table

<table>
<thead>
<tr>
<th>y in</th>
<th>x</th>
<th>inc</th>
<th>y out</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>-0.25</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>9</td>
<td>-3</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>9</td>
<td>-0.4</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>2.5</td>
<td>5</td>
</tr>
</tbody>
</table>
Example

Active edge list

<table>
<thead>
<tr>
<th>x</th>
<th>inc</th>
<th>y out</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.25</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>-3</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>-0.4</td>
<td>5</td>
</tr>
</tbody>
</table>
Example

Active edge list

<table>
<thead>
<tr>
<th>x</th>
<th>inc</th>
<th>y out</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.25</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>-3</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>-0.4</td>
<td>5</td>
</tr>
</tbody>
</table>

$y = 0$
Example

Active edge list

<table>
<thead>
<tr>
<th>x</th>
<th>inc</th>
<th>y out</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>-0.25</td>
<td>4</td>
</tr>
<tr>
<td>8.6</td>
<td>-0.4</td>
<td>5</td>
</tr>
</tbody>
</table>

$y = 1$
Example

Active edge list

<table>
<thead>
<tr>
<th>x</th>
<th>inc</th>
<th>y out</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>-0.25</td>
<td>4</td>
</tr>
<tr>
<td>8.6</td>
<td>-0.4</td>
<td>5</td>
</tr>
</tbody>
</table>

$y = 1$
Example

Active edge list

<table>
<thead>
<tr>
<th>x</th>
<th>inc</th>
<th>y out</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>-0.25</td>
<td>4</td>
</tr>
<tr>
<td>8.2</td>
<td>-0.4</td>
<td>5</td>
</tr>
</tbody>
</table>
Example

Active edge list

<table>
<thead>
<tr>
<th>x</th>
<th>inc</th>
<th>y out</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>-0.25</td>
<td>4</td>
</tr>
<tr>
<td>8.2</td>
<td>-0.4</td>
<td>5</td>
</tr>
</tbody>
</table>

y=2
Example

Active edge list

<table>
<thead>
<tr>
<th>x</th>
<th>inc</th>
<th>y out</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>-0.25</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2.5</td>
<td>5</td>
</tr>
<tr>
<td>7.8</td>
<td>-0.4</td>
<td>5</td>
</tr>
</tbody>
</table>
Example

Active edge list

<table>
<thead>
<tr>
<th>x</th>
<th>inc</th>
<th>y out</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>-0.25</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2.5</td>
<td>5</td>
</tr>
<tr>
<td>7.8</td>
<td>-0.4</td>
<td>5</td>
</tr>
</tbody>
</table>

$y=3$
Example

Active edge list

<table>
<thead>
<tr>
<th></th>
<th>inc</th>
<th>y out</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>2.5</td>
<td>5</td>
</tr>
<tr>
<td>7.4</td>
<td>-0.4</td>
<td>5</td>
</tr>
</tbody>
</table>
Example

Active edge list

<table>
<thead>
<tr>
<th>x</th>
<th>inc</th>
<th>y out</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>2.5</td>
<td>5</td>
</tr>
<tr>
<td>7.4</td>
<td>-0.4</td>
<td>5</td>
</tr>
</tbody>
</table>

$y=4$
Example

Active edge list

<table>
<thead>
<tr>
<th>x</th>
<th>inc</th>
<th>y out</th>
</tr>
</thead>
</table>

$y = 5$
OpenGL

OpenGL is optimised for implementation on hardware.

Hardware implementations do not work well with variable length lists.

So OpenGL enforces polygons to be convex. This means the active edge list always has 2 entries.

More complex polygons need to be tessellated into simple convex pieces.
Aliasing

Lines and polygons drawn with these algorithms tend to look jagged if the pixel size is too large.

This is another form of aliasing.
Aliasing

Lines and polygons drawn with these algorithms tend to look *jagged* if the pixel size is too large.

This is another form of *aliasing*. 
Antialiasing

There are two basic approaches to eliminating aliasing (antialiasing).

Prefiltering is computing exact pixel values geometrically rather than by sampling.

Postfiltering is taking samples at a higher resolution (supersampling) and then averaging.
Prefiltering

For each pixel, compute the amount occupied and set pixel value to that percentage.
Prefiltering

For each pixel, compute the amount occupied and set pixel value to that percentage.
Postfiltering

Draw the line at a higher resolution and average (supersampling).
Postfiltering

Draw the line at a higher resolution and average (supersampling)
Postfiltering

Draw the line at a higher resolution and average (supersampling).
Comparing

Prefiltering

Postfiltering
Weighted postfiltering

It is common to apply weights to the samples to favour values in the center of the pixel.

\[
\begin{array}{ccc}
\frac{1}{16} & \frac{1}{16} & \frac{1}{16} \\
\frac{1}{16} & \frac{1}{2} & \frac{1}{16} \\
\frac{1}{16} & \frac{1}{16} & \frac{1}{16}
\end{array}
\]
Stochastic sampling

Taking supersamples in a grid still tends to produce noticeably regular aliasing effects.

Adding small amounts of jitter to the sampled points makes aliasing effects appear as visual noise.
Adaptive Sampling

Supersampling in large areas of uniform colour is wasteful.

Supersampling is most useful in areas of major colour change.

Solution: Sample recursively, at finer levels of detail in areas with more colour variance.
Adaptive sampling

Samples
Adaptive sampling
Adaptive sampling
Antialiasing

Prefiltering is most accurate but requires more computation.

Postfiltering can be faster. Accuracy depends on how many samples are taken per pixel. More samples means larger memory usage.
OpenGL

// implementation dependant may not even do anything 😊

gl.glEnable(GL2.GL_LINE_SMOOTH);
gl.glHint(GL2.GL_LINE_SMOOTH_HINT,GL2.GL_NICEST);

// also requires alpha blending

gl.glEnable(GL2.GL_BLEND);
gl.glBlendFunc(GL2.GL_SRC_ALPHA,
              GL2.GL_ONE_MINUS_SRC_ALPHA);
// full-screen multi-sampling
GLCapabilities capabilities = new GLCapabilities();
capabilities.setNumSamples(4);
capabilities.setSampleBuffers(true);

// ...

gl.glEnable(GL.GL_MULTISAMPLE);