COMP 4161

NICTA Advanced Course

Advanced Topics in Software Verification

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Content

- Intro & motivation, getting started [1]
- Foundations & Principles
  - Lambda Calculus, natural deduction [1,2]
  - Higher Order Logic [3]
  - Term rewriting [4[a]]
- Proof & Specification Techniques
  - Inductively defined sets, rule induction [5]
  - Datatypes, recursion, induction [6, 7]
  - Code generation, type classes [7]
  - Hoare logic, proofs about programs, refinement [8[b],9[c],10]
  - Isar, locales [11[d],12]

[a] a1 due; [b] a2 due; [c] session break; [d] a3 due

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Automatic Proof and Disproof

➜ Sledgehammer: automatic proofs

➜ Quickcheck: counter example by testing

➜ Nipick: counter example by SAT

Based on slides by Jasmin Blanchette, Lukas Bulwahn, and Tobias Nipkow (TUM).

Automation

Dramatic improvements in fully automated proofs in the last 2 decades.

➜ First-order logic (ATP): Otter, Vampire, E, SPASS

➜ Propositional logic (SAT): MiniSAT, Chaff, RSat

➜ SAT modulo theory (SMT): CVC3, Yices, Z3

The key:

Efficient reasoning engines, and restricted logics.

Automation in Isabelle

1980s rule applications, write ML code

1990s simplifier, automatic provers (blast, auto), arithmetic

2000s embrace external tools, but don't trust them (ATP/SMT/SAT)

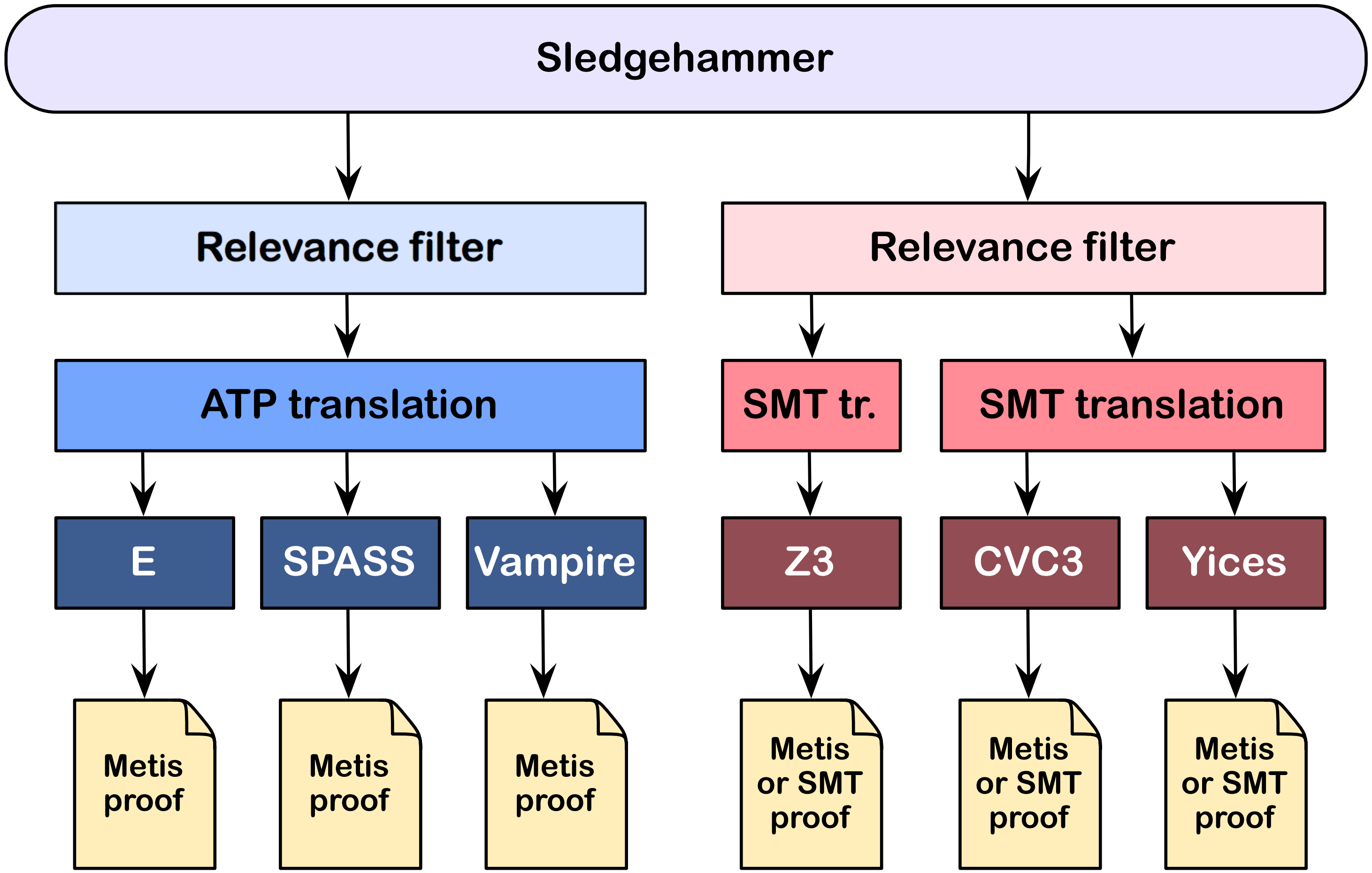
Sledgehammer

Slegehammer:

- Connects Isabelle with ATPs and SMT solvers:
  E, SPASS, Vampire, CVC3, Yices, Z3
- Simple invocation:
  - Users don't need to select or know facts
  - or ensure the problem is first-order
  - or know anything about the automated prover
- Exploits local parallelism and remote servers

DEMO: SLEDGEHAMMER

Sledgehammer Architecture



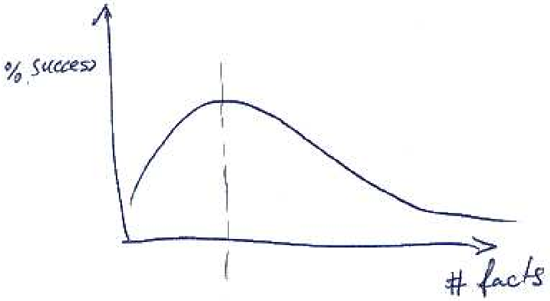
Fact Selection

Provers perform poorly if given 1000s of facts.

- ➜ Best number of facts depends on the prover
- ➜ Need to take care which facts we give them
- ➜ Idea: order facts by relevance, give top n to prover ($n = 250, 1000, \ldots$)

- ➜ Meng & Paulson method: lightweight, symbol-based filter

- ➜ Machine learning method:
  look at previous proofs to get a probability of relevance



From HOL to FOL

Source: higher-order, polymorphism, type classes

Target: first-order, untyped or simply-typed

- ➜ **First-order:**
  - ➜ SK combinators, λ-lifting
  - ➜ Explicit function application operator
- ➜ **Encode types:**
  - ➜ Monomorphise (generate multiple instances), or
  - ➜ Encode polymorphism on term level

Reconstruction

We don't want to trust the external provers.

Need to check/reconstruct proof.

➜ Re-find using Metis
  Usually fast and reliable (sometimes too slow)

➜ Rerun external prover for trusted replay
  Used for SMT. Re-runs prover each time!

➜ Recheck stored explicit external representation of proof
  Used for SMT, no need to re-run. Fragile.

➜ Recast into structured Isar proof
  Fast, experimental.

Judgement Day

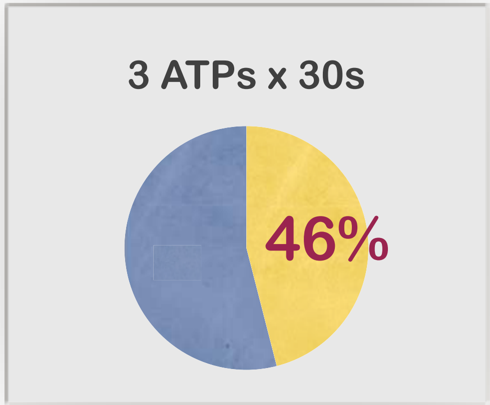
Evaluating Sledgehammer:

- ➜ 1240 goals out of 7 existing theories.
- ➜ How many can sledgehammer solve?

- ➜ **2010:** E, SPASS, Vampire (for 5-120s). 46%
  $ESV \times 5s \approx V \times 120s$
- ➜ **2011:** Add E-SInE, CVC2, Yices, Z3 (30s).
  $Z3 > V$
- ➜ **2012:** Better integration with SPASS. 64%
  SPASS best (small margin)
- ➜ **2013:** Machine learning for fact selection. 69%
  Improves a few percent across provers.

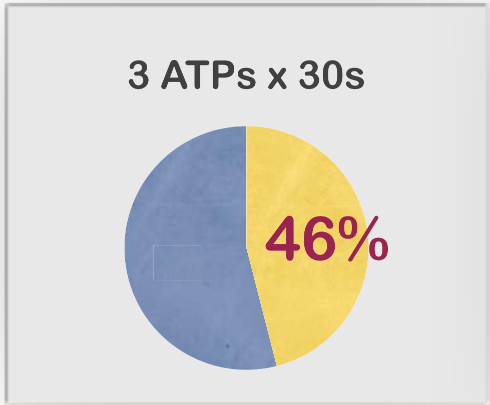
Evaluation

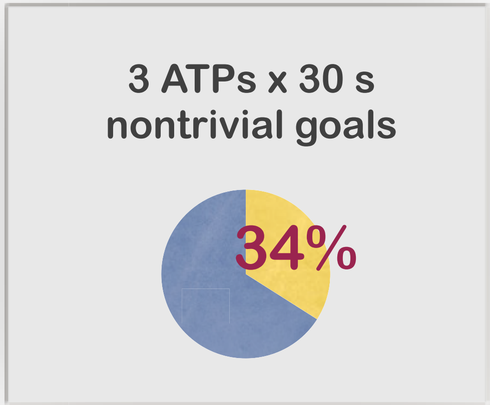
2010



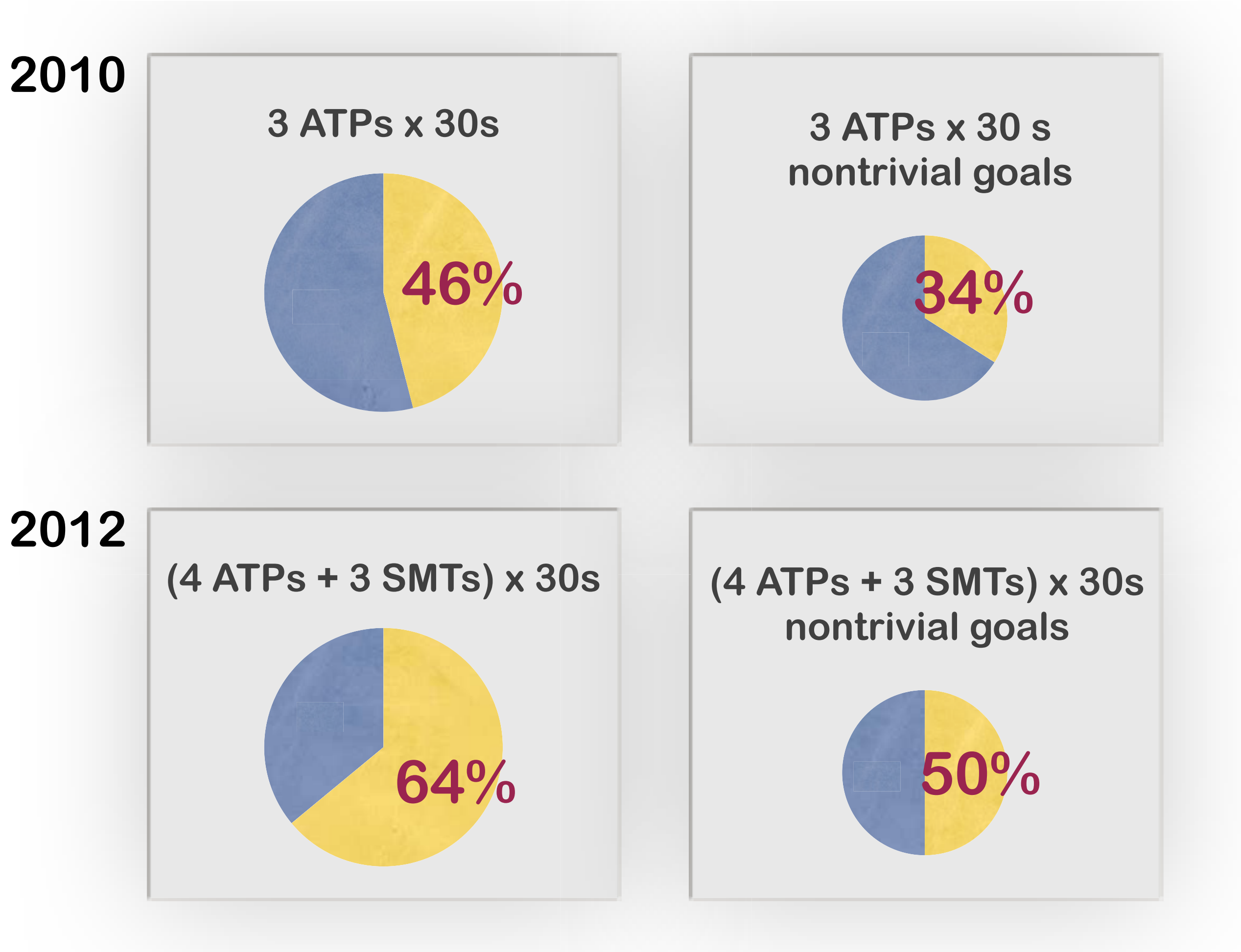
Evaluation

2010





Evaluation


2010
3 ATPs x 30s
46%
3 ATPs x 30 s
nontrivial goals
34%
2012
(4 ATPs + 3 SMTs) x 30s
64%
(4 ATPs + 3 SMTs) x 30s
nontrivial goals
50%

Sledgehammer rules!

Example application:

- Large Isabelle/HOL repository of algebras for modelling imperative programs (Kleene Algebra, Hoare logic, $\ldots$, ≈ 1000 lemmas)
- Intricate refinement and termination theorems
- Sledgehammer and Z3 automate algebraic proofs at textbook level.

"The integration of ATP, SMT, and Nitpick is for our purposes very very helpful." – G. Struth

DISPROOF

Theorem proving and testing

Testing can show only the presence of errors,
but not their absence. (Dijkstra)

Testing cannot prove theorems, but it can refute conjectures!

Sad facts of life:

- ➜ Most lemma statements are wrong the first time.
- ➜ Theorem proving is expensive as a debugging technique.

Find counter examples automatically!

Quickcheck

Lightweight validation by testing.

- ➜ Motivated by Haskell's QuickCheck
- ➜ Uses Isabelle's code generator
- ➜ Fast
- ➜ Runs in background, proves you wrong as you type.
  (current version: PG only, next release also in jEdit)

Quickcheck

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Covers a number of testing approaches:

➜ Random and exhausting testing.
➜ Smart test data generators.
➜ Narrowing-based (symbolic) testing.

Creates test data generators automatically.

DEMO: QUICKCHECK

Test generators for datatypes

Fast iteration in continuation-passing-style

datatype α list = Nil | Cons α (α list)

Test function:

$\text{test}_{\alpha\ list}$ P = P Nil *andalso* test_{α} (λx. $\text{test}_{\alpha\ list}$ (λxs. P (Cons x xs)))

Test generators for predicates

distinct xs $\Longrightarrow$ distinct (remove1 x xs)

Problem:

Exhaustive testing creates many useless test cases.

Solution:

Use definitions in precondition for smarter generator.

Only generate cases where *distinct xs* is true.

test-distinct$_{\alpha\ list}$ P = P Nil *andalso*

test$_{\alpha}$ (λx. test-distinct$_{\alpha\ list}$ (if x $\notin$ xs then (λxs. P (Cons x xs)) else True))

Use data flow analysis to figure out which variables must be computed and which generated.

Narrowing

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Symbolic execution with demand-driven refinement

➜ Test cases can contain variables

➜ If execution cannot proceed: instantiate with further symbolic terms

Pays off if large search spaces can be discarded:

distinct (Cons 1 (Cons 1 x))

False for any *x*, no further instantiations for x necessary.

Implementation:

Lazy execution with outer refinement loop.

Many re-computations, but fast.

Quickcheck Limitations

Only executable specifications!

➜ No equality on functions with infinite domain

➜ No axiomatic specifications

NITPICK

Finite model finder

- ➜ Based on SAT via Kodkod (backend of Alloy prover)
- ➜ Soundly approximates infinite types

Nitpick Successes

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- Algebraic methods
- C++ memory model
- Found soundness bugs in TPS and LEO-II

Fan mail:

"Last night I got stuck on a goal I was sure was a theorem. After 5–10 minutes I gave Nitpick a try, and within a few secs it had found a splendid counterexample—despite the mess of locales and type classes in the context!"

Demo: Nitpick

We have seen today ...

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- ➜ Proof: Sledgehammer
- ➜ Counter examples: Quickcheck
- ➜ Counter examples: Nitpick