

#### COMP 4161

#### Data61 Advanced Course

#### **Advanced Topics in Software Verification**

Gerwin Klein, June Andronick, Ramana Kumar, Miki Tanaka

# Binary Search (java.util.Arrays)



```
1:
      public static int binarySearch(int[] a, int key) {
2:
           int low = 0:
3:
           int high = a.length - 1;
4:
5:
           while (low <= high) {
6.
               int mid = (low + high) / 2;
               int midVal = a[mid];
7:
8.
9:
               if (midVal < key)
10:
                    low = mid + 1
11:
                else if (midVal > key)
                    high = mid -1;
12:
13:
                else
14:
                    return mid; // key found
15:
            3
16:
            return -(low + 1); // key not found.
17:
        }
```

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17:
        3
```

#### 6: int mid = (low + high) / 2;

```
http://googleresearch.blogspot.com/2006/06/
extra-extra-read-all-about-it-nearly.html
```

## Organisatorials



- When Mon 9:30 11:00 Thu 12:00 - 13:30
- Where Mon: Old Main Building 150 (K-K15-150) Thu: Central Lecture Block 8 (K-E19-105)

http://www.cse.unsw.edu.au/~cs4161/



#### The trustworthy systems verification team

→ Functional correctness and security of the seL4 microkernel Security ↔ Isabelle/HOL model ↔ Haskell model ↔ C code ↔ Binary



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Open Source http://sel4.systems https://cakeml.org



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#### We are always embarking on exciting new projects. We offer

- → summer student scholarship projects
- ➔ honours and PhD theses
- → research assistant and verification engineer positions



→ how to use a theorem prover



- ➔ how to use a theorem prover
- → background, how it works



- $\clubsuit$  how to use a theorem prover
- → background, how it works
- ➔ how to prove and specify



- $\rightarrow$  how to use a theorem prover
- → background, how it works
- → how to prove and specify
- → how to reason about programs



- → how to use a theorem prover
- → background, how it works
- → how to prove and specify
- → how to reason about programs

## Health Warning Theorem Proving is addictive

## Prerequisites



#### This is an advanced course. It assumes knowledge in

- ➔ Functional programming
- ➔ First-order formal logic

## Prerequisites



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- ➔ Functional programming
- ➔ First-order formal logic

The following program should make sense to you:

$$\begin{array}{lll} \mathsf{map} \ \mathsf{f} \ [] & = & [] \\ \mathsf{map} \ \mathsf{f} \ (\mathsf{x}{:}\mathsf{xs}) & = & \mathsf{f} \ \mathsf{x} : \ \mathsf{map} \ \mathsf{f} \ \mathsf{xs} \end{array}$$

### Prerequisites



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- → Functional programming
- ➔ First-order formal logic

The following program should make sense to you:

$$\begin{array}{ll} \mathsf{map} \ \mathsf{f} \ [] & = & [] \\ \mathsf{map} \ \mathsf{f} \ (\mathsf{x}:\mathsf{xs}) & = & \mathsf{f} \ \mathsf{x} : \ \mathsf{map} \ \mathsf{f} \ \mathsf{xs} \end{array}$$

You should be able to read and understand this formula:

$$\exists x. \ (P(x) \longrightarrow \forall x. \ P(x))$$



➔ Intro & motivation, getting started



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- → Foundations & Principles
  - Lambda Calculus, natural deduction
  - Higher Order Logic
  - Term rewriting



➔ Intro & motivation, getting started

- ➔ Foundations & Principles
  - Lambda Calculus, natural deduction
  - Higher Order Logic
  - Term rewriting
- ➔ Proof & Specification Techniques
  - Inductively defined sets, rule induction
  - Datatypes, recursion, induction
  - Hoare logic, proofs about programs, C verification
  - Writing Automated Proof Methods
  - Isar, codegen, typeclasses, locales



		Rough timeline
→	Intro & motivation, getting started	[today]
<b>→</b>	<ul> <li>Foundations &amp; Principles</li> <li>Lambda Calculus, natural deduction</li> <li>Higher Order Logic</li> <li>Term rewriting</li> </ul>	[1,2] [3 <sup>a</sup> ] [4]
<b>→</b>	<ul> <li>Proof &amp; Specification Techniques</li> <li>Inductively defined sets, rule induction</li> <li>Datatypes, recursion, induction</li> <li>Hoare logic, proofs about programs, C verification</li> <li>(mid-semester break)</li> <li>Writing Automated Proof Methods</li> <li>Isar, codegen, typeclasses, locales</li> </ul>	[5] [6, 7] [8 <sup>b</sup> ,9] [10] [11 <sup>c</sup> ,12]

<sup>a</sup>a1 due; <sup>b</sup>a2 due; <sup>c</sup>a3 due





→ attend lectures



- → attend lectures
- → try Isabelle early



- → attend lectures
- → try Isabelle early
- → redo all the demos alone



- → attend lectures
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- $\rightarrow$  redo all the demos alone
- ightarrow try the exercises/homework we give, when we do give some



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#### → DO NOT CHEAT

- Assignments and exams are take-home. This does NOT mean you can work in groups. Each submission is personal.
- For more info, see Plagiarism Policy<sup>a</sup>

<sup>a</sup> https://student.unsw.edu.au/plagiarism

### Credits



some material (in using-theorem-provers part) shamelessly stolen from



#### Tobias Nipkow, Larry Paulson, Markus Wenzel



David Basin, Burkhardt Wolff

Don't blame them, errors are ours



to prove

to prove

→ from Latin probare (test, approve, prove)



to prove

- → from Latin probare (test, approve, prove)
- → to learn or find out by experience (archaic)





#### to prove

- → from Latin probare (test, approve, prove)
- → to learn or find out by experience (archaic)
- → to establish the existence, truth, or validity of (by evidence or logic) prove a theorem, the charges were never proved in court



#### to prove

- → from Latin probare (test, approve, prove)
- → to learn or find out by experience (archaic)
- to establish the existence, truth, or validity of (by evidence or logic) prove a theorem, the charges were never proved in court

#### pops up everywhere

- ➔ politics (weapons of mass destruction)
- → courts (beyond reasonable doubt)
- → religion (god exists)
- → science (cold fusion works)

## What is a mathematical proof?



In mathematics, a proof is a demonstration that, given certain axioms, some statement of interest is necessarily true. (Wikipedia)

**Example:**  $\sqrt{2}$  is not rational.

Proof:

## What is a mathematical proof?



In mathematics, a proof is a demonstration that, given certain axioms, some statement of interest is necessarily true. (Wikipedia)

**Example:**  $\sqrt{2}$  is not rational.

Proof: assume there is  $r \in \mathbb{Q}$  such that  $r^2 = 2$ . Hence there are mutually prime p and q with  $r = \frac{p}{q}$ . Thus  $2q^2 = p^2$ , i.e.  $p^2$  is divisible by 2. 2 is prime, hence it also divides p, i.e. p = 2s. Substituting this into  $2q^2 = p^2$  and dividing by 2 gives  $q^2 = 2s^2$ . Hence, q is also divisible by 2. Contradiction. Qed.

## Nice, but..

 $\rightarrow$  still not rigorous enough for some

- what are the rules?
- what are the axioms?
- how big can the steps be?
- what is obvious or trivial?
- $\rightarrow$  informal language, easy to get wrong
- ➔ easy to miss something, easy to cheat



### Nice, but..

→ still not rigorous enough for some

- what are the rules?
- what are the axioms?
- how big can the steps be?
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- $\rightarrow$  easy to miss something, easy to cheat

**Theorem.** A cat has nine tails.

**Proof.** No cat has eight tails. Since one cat has one more tail than no cat, it must have nine tails.

### What is a formal proof?



A derivation in a formal calculus

### What is a formal proof?



A derivation in a formal calculus Example:  $A \land B \longrightarrow B \land A$  derivable in the following system Rules:  $\frac{X \in S}{S \vdash X}$  (assumption)  $\frac{S \cup \{X\} \vdash Y}{S \vdash X \longrightarrow Y}$  (impl)  $\frac{S \vdash X \quad S \vdash Y}{S \vdash X \land Y}$  (conjl)  $\frac{S \cup \{X, Y\} \vdash Z}{S \cup \{X \land Y\} \vdash Z}$  (conjE)

### What is a formal proof?

A derivation in a formal calculus **Example:**  $A \land B \longrightarrow B \land A$  derivable in the following system **Rules:**  $\frac{X \in S}{S \vdash X}$  (assumption)  $\frac{S \cup \{X\} \vdash Y}{S \vdash X \longrightarrow Y}$  (impl)  $\frac{S \vdash X \quad S \vdash Y}{S \vdash X \land Y} \text{ (conjl)} \quad \frac{S \cup \{X, Y\} \vdash Z}{S \cup \{X \land Y\} \vdash Z} \text{ (conjE)}$ Proof: 1.  $\{A, B\} \vdash B$ (by assumption) 2. 3.  $\{A, B\} \vdash A$ (by assumption)  $\{A, B\} \vdash B \land A$  (by conjl with 1 and 2)  $\{A \land B\} \vdash B \land A$  (by conjE with 3)  $\{\} \vdash A \land B \longrightarrow B \land A$  (by impl with 4) 4. 5

DATA

### What is a theorem prover?



#### Implementation of a formal logic on a computer.

- → fully automated (propositional logic)
- → automated, but not necessarily terminating (first order logic)
- → with automation, but mainly interactive (higher order logic)

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- → can deliver proofs

### What is a theorem prover?



#### Implementation of a formal logic on a computer.

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- ➔ based on rules and axioms
- → can deliver proofs

There are other (algorithmic) verification tools:

- → model checking, static analysis, ...
- ➔ usually do not deliver proofs
- → See COMP3153: Algorithmic Verification



➔ Analysing systems/programs thoroughly



- ➔ Analysing systems/programs thoroughly
- → Finding design and specification errors early



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- → Finding design and specification errors early
- → High assurance (mathematical, machine checked proof)



- ➔ Analysing systems/programs thoroughly
- → Finding design and specification errors early
- → High assurance (mathematical, machine checked proof)
- → it's not always easy
- → it's fun

### Main theorem proving system for this course





Isabelle

 $\label{eq:second}$  used here for applications, learning how to prove

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A generic interactive proof assistant



#### A generic interactive proof assistant

#### → generic:

not specialised to one particular logic (two large developments: HOL and ZF, will mainly use HOL)



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more than just yes/no, you can interactively guide the system



#### A generic interactive proof assistant

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not specialised to one particular logic (two large developments: HOL and ZF, will mainly use HOL)

#### → interactive:

more than just yes/no, you can interactively guide the system

#### → proof assistant:

helps to explore, find, and maintain proofs

### Why Isabelle?



➔ free

- ➔ widely used systems
- → active development
- $\rightarrow$  high expressiveness and automation
- $\rightarrow$  reasonably easy to use

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#### ➔ free

- → widely used systems
- → active development
- $\rightarrow$  high expressiveness and automation
- $\rightarrow$  reasonably easy to use
- → (and because we know it best ;-))



#### No, because:

① hardware could be faulty

- 1 hardware could be faulty
- $\ensuremath{\textcircled{}^{2}}$  operating system could be faulty

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- ① hardware could be faulty
- ② operating system could be faulty
- ③ implementation runtime system could be faulty

DATA



- 1 hardware could be faulty
- operating system could be faulty
- ③ implementation runtime system could be faulty
- ④ compiler could be faulty

DATA

- 1 hardware could be faulty
- ② operating system could be faulty
- 3 implementation runtime system could be faulty
- ④ compiler could be faulty
- ⑤ implementation could be faulty

DATA

- 1 hardware could be faulty
- ② operating system could be faulty
- 3 implementation runtime system could be faulty
- ④ compiler could be faulty
- ⑤ implementation could be faulty
- ⑥ logic could be inconsistent

DATA

- 1 hardware could be faulty
- ② operating system could be faulty
- ③ implementation runtime system could be faulty
- ④ compiler could be faulty
- ⑤ implementation could be faulty
- 6 logic could be inconsistent
- $\ensuremath{\mathbbm O}$  theorem could mean something else

No, but:

No, but: probability for

 $\rightarrow$  OS and H/W issues reduced by using different systems

#### No, but:

probability for

- $\clubsuit$  OS and H/W issues reduced by using different systems
- $\rightarrow$  runtime/compiler bugs reduced by using different compilers



probability for

- $\rightarrow$  OS and H/W issues reduced by using different systems
- $\rightarrow$  runtime/compiler bugs reduced by using different compilers
- $\rightarrow$  faulty implementation reduced by having the right prover architecture

DATA



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 $\boldsymbol{\Rightarrow}$  inconsistent logic reduced by implementing and analysing it



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DATA

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DATA

- $\boldsymbol{\rightarrow}$  inconsistent logic reduced by implementing and analysing it
- → wrong theorem reduced by expressive/intuitive logics

No guarantees, but assurance immensly higher than manual proof

Soundness architectures careful implementation

PVS

#### Soundness architectures

careful implementation

PVS

LCF approach, small proof kernel

HOL4 Isabelle

# If I prove it on the computer, it is correct, right?



careful implementation

LCF approach, small proof kernel

explicit proofs + proof checker

HOL4 Isabelle

PVS

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Coq Twelf Isabelle HOL4

### Meta Logic



#### Meta language:

The language used to talk about another language.

## Meta Logic



#### Meta language:

The language used to talk about another language.

### Examples:

English in a Spanish class, English in an English class

## Meta Logic



#### Meta language:

The language used to talk about another language.

### Examples:

English in a Spanish class, English in an English class

### Meta logic:

The logic used to formalize another logic

### Example:

Mathematics used to formalize derivations in formal logic

## Meta Logic – Example



Formulae:
$$F ::= V | F \rightarrow F | F \land F | False$$
Syntax: $V ::= [A - Z]$ Derivable: $S \vdash X X$  a formula, S a set of formulae

### Meta Logic – Example

-



Formulae: 
$$F ::= V | F \longrightarrow F | F \land F | Fall$$
  
Syntax:  $V ::= [A - Z]$   
Derivable:  $S \vdash X$  X a formula, S a set of formulae  
 $logic / meta logic$   
 $X \in S$   
 $S \vdash X$   $S \cup \{X\} \vdash Y$   
 $S \vdash X \longrightarrow Y$ 

 $\frac{S \vdash X \quad S \vdash Y}{S \vdash X \land Y} \qquad \frac{S \cup \{X, Y\} \vdash Z}{S \cup \{X \land Y\} \vdash Z}$ 

### Isabelle's Meta Logic



# $\bigwedge \implies \lambda$

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### **Syntax:** $\bigwedge x. F$ (*F* another meta level formula) in ASCII: !!x. F



### **Syntax:** $\bigwedge x. F$ (*F* another meta level formula) in ASCII: !!x. F

- ightarrow universal quantifier on the meta level
- ➔ used to denote parameters
- → example and more later





### **Syntax:** $A \Longrightarrow B$ (*A*, *B* other meta level formulae)

in ASCII: A ==> B

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**Syntax:**  $A \Longrightarrow B$  (A, B other meta level formulae) in ASCII:  $A \implies B$ 

Binds to the right:

$$A \Longrightarrow B \Longrightarrow C = A \Longrightarrow (B \Longrightarrow C)$$

Abbreviation:

$$\llbracket A; B \rrbracket \Longrightarrow C \quad = \quad A \Longrightarrow B \Longrightarrow C$$

 $\rightarrow$  read: A and B implies C

 $\rightarrow$  used to write down rules, theorems, and proof states



### **mathematics:** if x < 0 and y < 0, then x + y < 0

mathematics: if x < 0 and y < 0, then x + y < 0

formal logic:  $\vdash x < 0 \land y < 0 \longrightarrow x + y < 0$ variation:  $x < 0; y < 0 \vdash x + y < 0$ 



DATA

**mathematics:** if 
$$x < 0$$
 and  $y < 0$ , then  $x + y < 0$ 

variation:

formal logic:  $\vdash x < 0 \land y < 0 \longrightarrow x + y < 0$  $x < 0; y < 0 \vdash x + y < 0$ 

Isabelle: variation:

lemma "
$$x < 0 \land y < 0 \longrightarrow x + y < 0$$
"  
lemma " $[x < 0; y < 0] \implies x + y < 0$ "

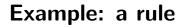


mathematics: if x < 0 and y < 0, then x + y < 0

variation:

formal logic:  $\vdash x < 0 \land y < 0 \longrightarrow x + y < 0$  $x < 0; y < 0 \vdash x + y < 0$ 

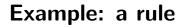
Isabelle: variation: variation: lemma " $x < 0 \land y < 0 \longrightarrow x + y < 0$ " lemma "  $[x < 0; y < 0] \implies x + y < 0$ " lemma assumes "x < 0" and "y < 0" shows "x + y < 0"





 $\frac{X \quad Y}{X \wedge Y}$ logic:

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 $\frac{X \quad Y}{X \wedge Y}$ logic:

 $\frac{S \vdash X \quad S \vdash Y}{S \vdash X \land Y}$ 

variation:

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### Example: a rule



logic: 
$$\frac{X Y}{X \wedge Y}$$

variation:

$$\frac{S \vdash X \quad S \vdash Y}{S \vdash X \land Y}$$

Isabelle: 
$$\llbracket X; Y \rrbracket \Longrightarrow X \land Y$$

# Example: a rule with nested implication



 $\begin{array}{ccc} X & Y \\ \vdots & \vdots \\ \underline{X \lor Y} & \underline{Z} & \underline{Z} \end{array}$ 

logic:

# Example: a rule with nested implication



logic:

variation:

 $\frac{S \cup \{X\} \vdash Z \quad S \cup \{Y\} \vdash Z}{S \cup \{X \lor Y\} \vdash Z}$ 

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# Example: a rule with nested implication



$$\frac{\begin{array}{ccc} X & Y \\ \vdots & \vdots \\ X \lor Y & Z & Z \\ Z \end{array}$$

logic:

$$\frac{S \cup \{X\} \vdash Z \quad S \cup \{Y\} \vdash Z}{S \cup \{X \lor Y\} \vdash Z}$$

variation:

**Isabelle:** 
$$[\![X \lor Y; X \Longrightarrow Z; Y \Longrightarrow Z]\!] \Longrightarrow Z$$

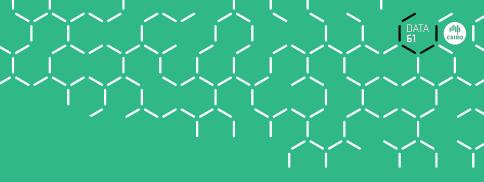


# **Syntax:** $\lambda x. F$ (*F* another meta level formula) in ASCII: % x. F



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- ➔ lambda abstraction
- ➔ used for functions in object logics
- → used to encode bound variables in object logics
- ➔ more about this in the next lecture



# **Enough Theory!**

Getting started with Isabelle



Isabelle - generic, interactive theorem prover



### Isabelle – generic, interactive theorem prover Standard ML – logic implemented as ADT



HOL, ZF – object-logics

Isabelle – generic, interactive theorem prover Standard ML – logic implemented as ADT



Prover IDE (jEdit) – user interface

HOL, ZF – object-logics

**Isabelle** – generic, interactive theorem prover

Standard ML - logic implemented as ADT



Prover IDE (jEdit) – user interface

HOL, ZF – object-logics

**Isabelle** – generic, interactive theorem prover

Standard ML – logic implemented as ADT

User can access all layers!

## System Requirements



- → Linux, Windows, or MacOS X (10.8 +)
- → Standard ML (PolyML implementation)
- → Java (for jEdit)

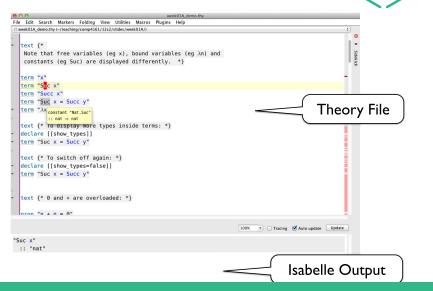
Premade packages for Linux, Mac, and Windows + info on: http://mirror.cse.unsw.edu.au/pub/isabelle/

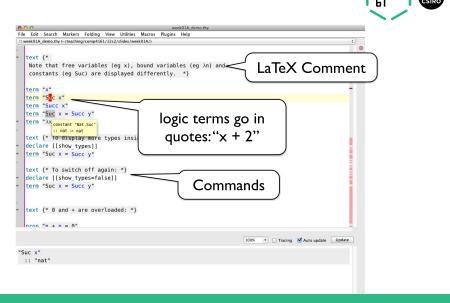
## Documentation

Available from http://isabelle.in.tum.de

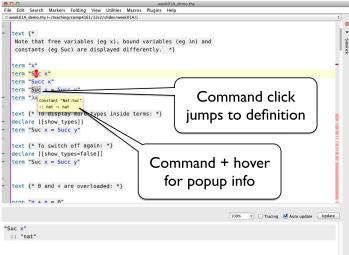
- → Learning Isabelle
  - Tutorial on Isabelle/HOL (LNCS 2283)
  - Tutorial on Isar
  - Tutorial on Locales
- ➔ Reference Manuals
  - Isabelle/Isar Reference Manual
  - Isabelle Reference Manual
  - Isabelle System Manual
- ➔ Reference Manuals for Object-Logics

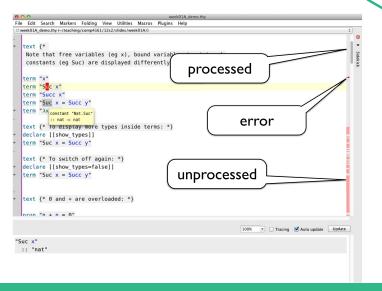
week01A_demo.thy (~/teaching/comp4161/12s2/slides/week01A/)	\$
text {*	
Note that free variables (eq x), bound variables (eq $\lambda$ n) and	Si Si
constants (eg Suc) are displayed differently. *}	Sidekick
	*
term "x"	-
term "Suc x"	
term "Succ x"	
term "Suc x = Succ y"	
term "Xx constant "Nat.Suc"	
:: nat ⇒ nat	
text {* To display more types inside terms: *}	
declare [[show types]]	
term "Suc x = Succ y"	
text {* To switch off again: *}	
declare [[show types=false]]	
term "Suc x = Succ y"	
text {* 0 and + are overloaded: *}	
nron "n + n = $\theta$ "	
100% × 🗆 Ti	racing 🥑 Auto update 🛛 Update
uc x"	
:: "nat"	

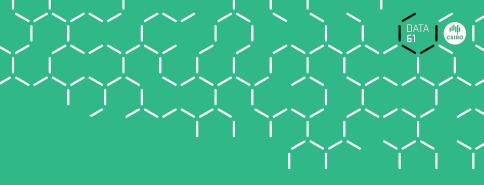












# Demo

### Exercises



- Download and install Isabelle from http://mirror.cse.unsw.edu.au/pub/isabelle/
- → Step through the demo files from the lecture web page
- → Write your own theory file, look at some theorems in the library, try 'find\_theorems'
- ➔ How many theorems can help you if you need to prove something containing the term "Suc(Suc x)"?
- → What is the name of the theorem for associativity of addition of natural numbers in the library?