

COMP 4161

Data61 Advanced Course

Advanced Topics in Software Verification

Gerwin Klein, June Andronick, Ramana Kumar, Miki Tanaka

Binary Search (java.util.Arrays)



```
1:
      public static int binarySearch(int[] a, int key) {
2:
           int low = 0:
3:
           int high = a.length - 1;
4:
5:
           while (low <= high) {
6.
               int mid = (low + high) / 2;
               int midVal = a[mid];
7:
8.
9:
               if (midVal < key)
10:
                    low = mid + 1
11:
                else if (midVal > key)
                    high = mid -1;
12:
13:
                else
14:
                    return mid; // key found
15:
            3
16:
            return -(low + 1); // key not found.
17:
        }
```

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                else
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                    return mid; // key found
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            3
16:
            return -(low + 1); // key not found.
17:
        3
```

6: int mid = (low + high) / 2;

```
http://googleresearch.blogspot.com/2006/06/
extra-extra-read-all-about-it-nearly.html
```

Organisatorials



- When Mon 9:30 11:00 Thu 12:00 - 13:30
- Where Mon: Old Main Building 150 (K-K15-150) Thu: Central Lecture Block 8 (K-E19-105)

http://www.cse.unsw.edu.au/~cs4161/



The trustworthy systems verification team

→ Functional correctness and security of the seL4 microkernel Security ↔ Isabelle/HOL model ↔ Haskell model ↔ C code ↔ Binary



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- → 10 000 LOC / 500 000 lines of proof script; about 25 person years of effort



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Open Source http://sel4.systems https://cakeml.org



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Open Source http://sel4.systems https://cakeml.org

We are always embarking on exciting new projects. We offer

- → summer student scholarship projects
- ➔ honours and PhD theses
- → research assistant and verification engineer positions



→ how to use a theorem prover



- ➔ how to use a theorem prover
- → background, how it works



- \clubsuit how to use a theorem prover
- → background, how it works
- ➔ how to prove and specify



- \rightarrow how to use a theorem prover
- → background, how it works
- → how to prove and specify
- → how to reason about programs



- → how to use a theorem prover
- → background, how it works
- → how to prove and specify
- → how to reason about programs

Health Warning Theorem Proving is addictive

Prerequisites



This is an advanced course. It assumes knowledge in

- ➔ Functional programming
- ➔ First-order formal logic

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- ➔ First-order formal logic

The following program should make sense to you:

$$\begin{array}{lll} \mathsf{map} \ \mathsf{f} \ [] & = & [] \\ \mathsf{map} \ \mathsf{f} \ (\mathsf{x}{:}\mathsf{xs}) & = & \mathsf{f} \ \mathsf{x} : \ \mathsf{map} \ \mathsf{f} \ \mathsf{xs} \end{array}$$

Prerequisites



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- ➔ First-order formal logic

The following program should make sense to you:

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You should be able to read and understand this formula:

$$\exists x. \ (P(x) \longrightarrow \forall x. \ P(x))$$



➔ Intro & motivation, getting started



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- → Foundations & Principles
 - Lambda Calculus, natural deduction
 - Higher Order Logic
 - Term rewriting



➔ Intro & motivation, getting started

- ➔ Foundations & Principles
 - Lambda Calculus, natural deduction
 - Higher Order Logic
 - Term rewriting
- ➔ Proof & Specification Techniques
 - Inductively defined sets, rule induction
 - Datatypes, recursion, induction
 - Hoare logic, proofs about programs, C verification
 - Writing Automated Proof Methods
 - Isar, codegen, typeclasses, locales



		Rough timeline
→	Intro & motivation, getting started	[today]
→	 Foundations & Principles Lambda Calculus, natural deduction Higher Order Logic Term rewriting 	[1,2] [3 ^a] [4]
→	 Proof & Specification Techniques Inductively defined sets, rule induction Datatypes, recursion, induction Hoare logic, proofs about programs, C verification (mid-semester break) Writing Automated Proof Methods Isar, codegen, typeclasses, locales 	[5] [6, 7] [8 ^b ,9] [10] [11 ^c ,12]

^aa1 due; ^ba2 due; ^ca3 due





→ attend lectures



- → attend lectures
- → try Isabelle early



- → attend lectures
- → try Isabelle early
- → redo all the demos alone



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- ightarrow try the exercises/homework we give, when we do give some



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→ DO NOT CHEAT

- Assignments and exams are take-home. This does NOT mean you can work in groups. Each submission is personal.
- For more info, see Plagiarism Policy^a

^a https://student.unsw.edu.au/plagiarism

Credits



some material (in using-theorem-provers part) shamelessly stolen from



Tobias Nipkow, Larry Paulson, Markus Wenzel



David Basin, Burkhardt Wolff

Don't blame them, errors are ours



to prove

to prove

→ from Latin probare (test, approve, prove)



to prove

- → from Latin probare (test, approve, prove)
- → to learn or find out by experience (archaic)





to prove

- → from Latin probare (test, approve, prove)
- → to learn or find out by experience (archaic)
- → to establish the existence, truth, or validity of (by evidence or logic) prove a theorem, the charges were never proved in court



to prove

- → from Latin probare (test, approve, prove)
- → to learn or find out by experience (archaic)
- to establish the existence, truth, or validity of (by evidence or logic) prove a theorem, the charges were never proved in court

pops up everywhere

- ➔ politics (weapons of mass destruction)
- → courts (beyond reasonable doubt)
- → religion (god exists)
- → science (cold fusion works)

What is a mathematical proof?



In mathematics, a proof is a demonstration that, given certain axioms, some statement of interest is necessarily true. (Wikipedia)

Example: $\sqrt{2}$ is not rational.

Proof:

What is a mathematical proof?



In mathematics, a proof is a demonstration that, given certain axioms, some statement of interest is necessarily true. (Wikipedia)

Example: $\sqrt{2}$ is not rational.

Proof: assume there is $r \in \mathbb{Q}$ such that $r^2 = 2$. Hence there are mutually prime p and q with $r = \frac{p}{q}$. Thus $2q^2 = p^2$, i.e. p^2 is divisible by 2. 2 is prime, hence it also divides p, i.e. p = 2s. Substituting this into $2q^2 = p^2$ and dividing by 2 gives $q^2 = 2s^2$. Hence, q is also divisible by 2. Contradiction. Qed.

Nice, but..

 \rightarrow still not rigorous enough for some

- what are the rules?
- what are the axioms?
- how big can the steps be?
- what is obvious or trivial?
- \rightarrow informal language, easy to get wrong
- ➔ easy to miss something, easy to cheat



Nice, but..

→ still not rigorous enough for some

- what are the rules?
- what are the axioms?
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Theorem. A cat has nine tails.

Proof. No cat has eight tails. Since one cat has one more tail than no cat, it must have nine tails.

What is a formal proof?



A derivation in a formal calculus

What is a formal proof?



A derivation in a formal calculus Example: $A \land B \longrightarrow B \land A$ derivable in the following system Rules: $\frac{X \in S}{S \vdash X}$ (assumption) $\frac{S \cup \{X\} \vdash Y}{S \vdash X \longrightarrow Y}$ (impl) $\frac{S \vdash X \quad S \vdash Y}{S \vdash X \land Y}$ (conjl) $\frac{S \cup \{X, Y\} \vdash Z}{S \cup \{X \land Y\} \vdash Z}$ (conjE)

What is a formal proof?

A derivation in a formal calculus **Example:** $A \land B \longrightarrow B \land A$ derivable in the following system **Rules:** $\frac{X \in S}{S \vdash X}$ (assumption) $\frac{S \cup \{X\} \vdash Y}{S \vdash X \longrightarrow Y}$ (impl) $\frac{S \vdash X \quad S \vdash Y}{S \vdash X \land Y} \text{ (conjl)} \quad \frac{S \cup \{X, Y\} \vdash Z}{S \cup \{X \land Y\} \vdash Z} \text{ (conjE)}$ Proof: 1. $\{A, B\} \vdash B$ (by assumption) 2. 3. $\{A, B\} \vdash A$ (by assumption) $\{A, B\} \vdash B \land A$ (by conjl with 1 and 2) $\{A \land B\} \vdash B \land A$ (by conjE with 3) $\{\} \vdash A \land B \longrightarrow B \land A$ (by impl with 4) 4. 5

DATA

What is a theorem prover?



Implementation of a formal logic on a computer.

- → fully automated (propositional logic)
- → automated, but not necessarily terminating (first order logic)
- → with automation, but mainly interactive (higher order logic)

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- ➔ based on rules and axioms
- → can deliver proofs

What is a theorem prover?



Implementation of a formal logic on a computer.

- → fully automated (propositional logic)
- → automated, but not necessarily terminating (first order logic)
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- ➔ based on rules and axioms
- → can deliver proofs

There are other (algorithmic) verification tools:

- → model checking, static analysis, ...
- ➔ usually do not deliver proofs
- → See COMP3153: Algorithmic Verification



➔ Analysing systems/programs thoroughly



- ➔ Analysing systems/programs thoroughly
- → Finding design and specification errors early



- ➔ Analysing systems/programs thoroughly
- → Finding design and specification errors early
- → High assurance (mathematical, machine checked proof)



- ➔ Analysing systems/programs thoroughly
- → Finding design and specification errors early
- → High assurance (mathematical, machine checked proof)
- → it's not always easy
- → it's fun

Main theorem proving system for this course





Isabelle

 $\label{eq:second}$ used here for applications, learning how to prove

DATA 61

A generic interactive proof assistant



A generic interactive proof assistant

→ generic:

not specialised to one particular logic (two large developments: HOL and ZF, will mainly use HOL)



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→ interactive:

more than just yes/no, you can interactively guide the system



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→ interactive:

more than just yes/no, you can interactively guide the system

→ proof assistant:

helps to explore, find, and maintain proofs

Why Isabelle?



➔ free

- ➔ widely used systems
- → active development
- \rightarrow high expressiveness and automation
- \rightarrow reasonably easy to use

Why Isabelle?



➔ free

- → widely used systems
- → active development
- \rightarrow high expressiveness and automation
- \rightarrow reasonably easy to use
- → (and because we know it best ;-))



No, because:

① hardware could be faulty

- 1 hardware could be faulty
- $\ensuremath{\textcircled{}^{2}}$ operating system could be faulty

DATA



- ① hardware could be faulty
- ② operating system could be faulty
- ③ implementation runtime system could be faulty

DATA



- 1 hardware could be faulty
- operating system could be faulty
- ③ implementation runtime system could be faulty
- ④ compiler could be faulty

DATA

- 1 hardware could be faulty
- ② operating system could be faulty
- 3 implementation runtime system could be faulty
- ④ compiler could be faulty
- ⑤ implementation could be faulty

DATA

- 1 hardware could be faulty
- ② operating system could be faulty
- 3 implementation runtime system could be faulty
- ④ compiler could be faulty
- ⑤ implementation could be faulty
- ⑥ logic could be inconsistent

DATA

- 1 hardware could be faulty
- ② operating system could be faulty
- ③ implementation runtime system could be faulty
- ④ compiler could be faulty
- ⑤ implementation could be faulty
- 6 logic could be inconsistent
- $\ensuremath{\mathbbm O}$ theorem could mean something else

No, but:

No, but: probability for

 \rightarrow OS and H/W issues reduced by using different systems

No, but:

probability for

- \clubsuit OS and H/W issues reduced by using different systems
- \rightarrow runtime/compiler bugs reduced by using different compilers



probability for

- \rightarrow OS and H/W issues reduced by using different systems
- \rightarrow runtime/compiler bugs reduced by using different compilers
- \rightarrow faulty implementation reduced by having the right prover architecture

DATA



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DATA

 $\boldsymbol{\Rightarrow}$ inconsistent logic reduced by implementing and analysing it



probability for

- \rightarrow OS and H/W issues reduced by using different systems
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DATA

- $\boldsymbol{\rightarrow}$ inconsistent logic reduced by implementing and analysing it
- \rightarrow wrong theorem reduced by expressive/intuitive logics

No, but:

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DATA

- $\boldsymbol{\rightarrow}$ inconsistent logic reduced by implementing and analysing it
- → wrong theorem reduced by expressive/intuitive logics

No guarantees, but assurance immensly higher than manual proof

Soundness architectures careful implementation

PVS

Soundness architectures

careful implementation

PVS

LCF approach, small proof kernel

HOL4 Isabelle

If I prove it on the computer, it is correct, right?



careful implementation

LCF approach, small proof kernel

explicit proofs + proof checker

HOL4 Isabelle

PVS

DATA

Coq Twelf Isabelle HOL4

Meta Logic



Meta language:

The language used to talk about another language.

Meta Logic



Meta language:

The language used to talk about another language.

Examples:

English in a Spanish class, English in an English class

Meta Logic



Meta language:

The language used to talk about another language.

Examples:

English in a Spanish class, English in an English class

Meta logic:

The logic used to formalize another logic

Example:

Mathematics used to formalize derivations in formal logic

Meta Logic – Example



Formulae:
$$F ::= V | F \rightarrow F | F \land F | False$$
Syntax: $V ::= [A - Z]$ Derivable: $S \vdash X X$ a formula, S a set of formulae

Meta Logic – Example

-



Formulae:
$$F ::= V | F \longrightarrow F | F \land F | Fall$$

Syntax: $V ::= [A - Z]$
Derivable: $S \vdash X$ X a formula, S a set of formulae
 $logic / meta logic$
 $X \in S$
 $S \vdash X$ $S \cup \{X\} \vdash Y$
 $S \vdash X \longrightarrow Y$

 $\frac{S \vdash X \quad S \vdash Y}{S \vdash X \land Y} \qquad \frac{S \cup \{X, Y\} \vdash Z}{S \cup \{X \land Y\} \vdash Z}$

Isabelle's Meta Logic



$\bigwedge \implies \lambda$

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Syntax: $\bigwedge x. F$ (*F* another meta level formula) in ASCII: !!x. F



Syntax: $\bigwedge x. F$ (*F* another meta level formula) in ASCII: !!x. F

- ightarrow universal quantifier on the meta level
- ➔ used to denote parameters
- → example and more later





Syntax: $A \Longrightarrow B$ (*A*, *B* other meta level formulae)

in ASCII: A ==> B

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Syntax: $A \Longrightarrow B$ (A, B other meta level formulae) in ASCII: $A \implies B$

Binds to the right:

$$A \Longrightarrow B \Longrightarrow C = A \Longrightarrow (B \Longrightarrow C)$$

Abbreviation:

$$\llbracket A; B \rrbracket \Longrightarrow C \quad = \quad A \Longrightarrow B \Longrightarrow C$$

 \rightarrow read: A and B implies C

 \rightarrow used to write down rules, theorems, and proof states



mathematics: if x < 0 and y < 0, then x + y < 0

mathematics: if x < 0 and y < 0, then x + y < 0

formal logic: $\vdash x < 0 \land y < 0 \longrightarrow x + y < 0$ variation: $x < 0; y < 0 \vdash x + y < 0$



DATA

mathematics: if
$$x < 0$$
 and $y < 0$, then $x + y < 0$

variation:

formal logic: $\vdash x < 0 \land y < 0 \longrightarrow x + y < 0$ $x < 0; y < 0 \vdash x + y < 0$

Isabelle: variation:

lemma "
$$x < 0 \land y < 0 \longrightarrow x + y < 0$$
"
lemma " $[x < 0; y < 0] \implies x + y < 0$ "

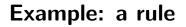


mathematics: if x < 0 and y < 0, then x + y < 0

variation:

formal logic: $\vdash x < 0 \land y < 0 \longrightarrow x + y < 0$ $x < 0; y < 0 \vdash x + y < 0$

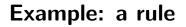
Isabelle: variation: variation: lemma " $x < 0 \land y < 0 \longrightarrow x + y < 0$ " lemma " $[x < 0; y < 0] \implies x + y < 0$ " lemma assumes "x < 0" and "y < 0" shows "x + y < 0"





 $\frac{X \quad Y}{X \wedge Y}$ logic:

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 $\frac{X \quad Y}{X \wedge Y}$ logic:

 $\frac{S \vdash X \quad S \vdash Y}{S \vdash X \land Y}$

variation:

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Example: a rule



logic:
$$\frac{X Y}{X \wedge Y}$$

variation:

$$\frac{S \vdash X \quad S \vdash Y}{S \vdash X \land Y}$$

Isabelle:
$$\llbracket X; Y \rrbracket \Longrightarrow X \land Y$$

Example: a rule with nested implication



 $\begin{array}{ccc} X & Y \\ \vdots & \vdots \\ \underline{X \lor Y} & \underline{Z} & \underline{Z} \end{array}$

logic:

Example: a rule with nested implication



logic:

variation:

 $\frac{S \cup \{X\} \vdash Z \quad S \cup \{Y\} \vdash Z}{S \cup \{X \lor Y\} \vdash Z}$

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Example: a rule with nested implication



$$\frac{\begin{array}{ccc} X & Y \\ \vdots & \vdots \\ X \lor Y & Z & Z \\ Z \end{array}$$

logic:

$$\frac{S \cup \{X\} \vdash Z \quad S \cup \{Y\} \vdash Z}{S \cup \{X \lor Y\} \vdash Z}$$

variation:

Isabelle:
$$[\![X \lor Y; X \Longrightarrow Z; Y \Longrightarrow Z]\!] \Longrightarrow Z$$

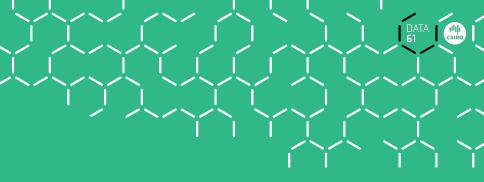


Syntax: $\lambda x. F$ (*F* another meta level formula) in ASCII: % x. F



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- ➔ lambda abstraction
- ➔ used for functions in object logics
- → used to encode bound variables in object logics
- ➔ more about this in the next lecture



Enough Theory!

Getting started with Isabelle



Isabelle - generic, interactive theorem prover



Isabelle – generic, interactive theorem prover Standard ML – logic implemented as ADT



HOL, ZF – object-logics

Isabelle – generic, interactive theorem prover Standard ML – logic implemented as ADT



Prover IDE (jEdit) – user interface

HOL, ZF – object-logics

Isabelle – generic, interactive theorem prover

Standard ML - logic implemented as ADT



Prover IDE (jEdit) – user interface

HOL, ZF – object-logics

Isabelle – generic, interactive theorem prover

Standard ML – logic implemented as ADT

User can access all layers!

System Requirements



- → Linux, Windows, or MacOS X (10.8 +)
- → Standard ML (PolyML implementation)
- → Java (for jEdit)

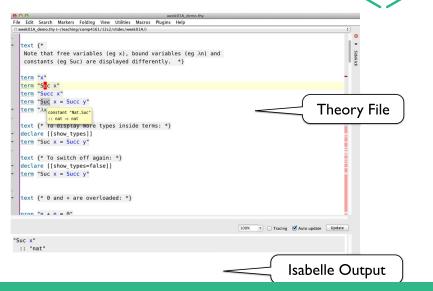
Premade packages for Linux, Mac, and Windows + info on: http://mirror.cse.unsw.edu.au/pub/isabelle/

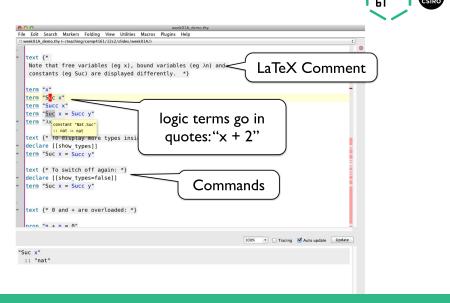
Documentation

Available from http://isabelle.in.tum.de

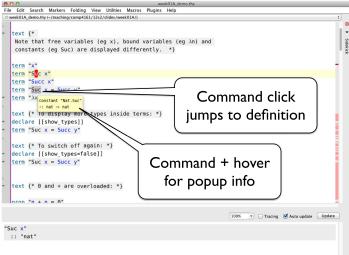
- → Learning Isabelle
 - Tutorial on Isabelle/HOL (LNCS 2283)
 - Tutorial on Isar
 - Tutorial on Locales
- ➔ Reference Manuals
 - Isabelle/Isar Reference Manual
 - Isabelle Reference Manual
 - Isabelle System Manual
- ➔ Reference Manuals for Object-Logics

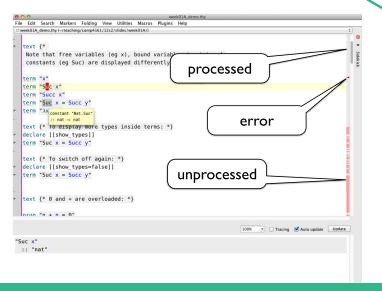
week01A_demo.thy (~/teaching/comp4161/12s2/slides/week01A/)	\$
text {*	
Note that free variables (eq x), bound variables (eq λ n) and	Si Si
constants (eg Suc) are displayed differently. *}	Sidekick
	*
term "x"	-
term "Suc x"	
term "Succ x"	
term "Suc x = Succ y"	
term "Xx constant "Nat.Suc"	
:: nat ⇒ nat	
text {* To display more types inside terms: *}	
declare [[show types]]	
term "Suc x = Succ y"	
text {* To switch off again: *}	
declare [[show types=false]]	
term "Suc x = Succ y"	
text {* 0 and + are overloaded: *}	
nron "n + n = θ "	
100% × 🗆 Ti	racing 🥑 Auto update 🛛 Update
uc x"	
:: "nat"	

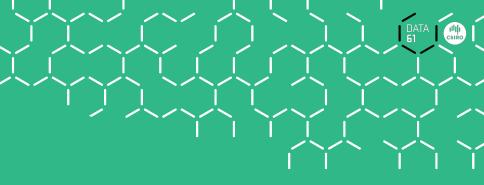












Demo

Exercises



- Download and install Isabelle from http://mirror.cse.unsw.edu.au/pub/isabelle/
- → Step through the demo files from the lecture web page
- → Write your own theory file, look at some theorems in the library, try 'find_theorems'
- ➔ How many theorems can help you if you need to prove something containing the term "Suc(Suc x)"?
- → What is the name of the theorem for associativity of addition of natural numbers in the library?