

COMP4161 S2/2014
 Advanced Topics in Software Verification
 Exam Solution

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1 Lambda Calculus

Consider the term $\lambda a. b. b (\lambda x. x a)$

- (a) What is its $\beta\eta$ -normal form?

Solution:

This term is already in $\beta\eta$ -normal form.

- (b) What is its type?

Solution:

$$\alpha \Rightarrow (((\alpha \Rightarrow \beta) \Rightarrow \beta) \Rightarrow \gamma) \Rightarrow \gamma$$

- (c) Provide a step-by-step derivation of its type.

Solution:

Let $\Gamma = [a \leftarrow \alpha, b \leftarrow ((\alpha \Rightarrow \beta) \Rightarrow \beta) \Rightarrow \gamma]$ and $\Gamma' = \Gamma[x \leftarrow \alpha \Rightarrow \beta]$

$$\frac{\frac{\frac{\frac{\frac{\Gamma' \vdash x :: \alpha \Rightarrow \beta}{\text{Var}} \quad \frac{\Gamma' \vdash a :: \alpha}{\text{Var}}}{\text{App}}}{\Gamma' \vdash (x a) :: \beta} \quad \frac{\Gamma \vdash b :: ((\alpha \Rightarrow \beta) \Rightarrow \beta) \Rightarrow \gamma}{\text{Var}} \quad \frac{\Gamma \vdash (\lambda x. x a) :: (\alpha \Rightarrow \beta) \Rightarrow \beta}{\text{Abs}}}{\Gamma \vdash b (\lambda x. x a) :: \gamma} \quad \text{App}}{\boxed{\vdash} \lambda a. b. b (\lambda x. x a) :: \alpha \Rightarrow (((\alpha \Rightarrow \beta) \Rightarrow \beta) \Rightarrow \gamma) \Rightarrow \gamma} \quad \text{Abs (x2)}$$

2 Induction

See the Isabelle theory.

3 C Verification

See the Isabelle theory.