# COMP9444
# Neural Networks and Deep Learning
# 10. Reinforcement Learning






## Outline

- Reinforcement Learning vs. Supervised Learning
- Models of Optimality
- Exploration vs. Exploitation
- Value Function Learning
  - Q-Learning
  - TD-Learning
- Policy Learning
  - Evolution Strategies
  - Policy Gradients
- Actor-Critic






## Supervised Learning

Recall: Supervised Learning

- We have a training set and a test set, each consisting of a set of examples. For each example, a number of input attributes and a target attribute are specified.
- The aim is to predict the target attribute, based on the input attributes.
- Various learning paradigms are available:
  - Decision Trees
  - Neural Networks
  - SVM
  - .. others ..






## Learning of Actions

Supervised Learning can also be used to learn Actions, if we construct a training set of situation-action pairs (called Behavioral Cloning).

However, there are many applications for which it is difficult, inappropriate, or even impossible to provide a "training set"

- optimal control
  - mobile robots, pole balancing, flying a helicopter
- resource allocation
  - job shop scheduling, mobile phone channel allocation
- mix of allocation and control
  - elevator control, backgammon

# Reinforcement Learning Framework

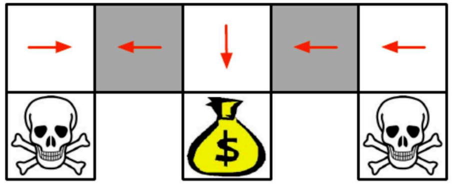
- An agent interacts with its environment.
- There is a set $\mathcal{S}$ of *states* and a set $\mathcal{A}$ of *actions*.
- At each time step $t$, the agent is in some state $s_t$. It must choose an action $a_t$, whereupon it goes into state $s_{t+1} = \delta(s_t, a_t)$ and receives reward $r_t = \mathcal{R}(s_t, a_t)$
- Agent has a *policy* $\pi : \mathcal{S} \to \mathcal{A}$. We aim to find an optimal policy $\pi^*$ which maximizes the cumulative reward.
- In general, $\delta$, $\mathcal{R}$ and $\pi$ can be multi-valued, with a random element, in which case we write them as probability distributions

$$\delta(s_{t+1} = s \,|\, s_t, a_t) \quad \mathcal{R}(r_t = r \,|\, s_t, a_t) \quad \pi(a_t = a \,|\, s_t)$$





# Probabilistic Policies



There are some environments in which any deterministic agent will perform very poorly, and the optimal (reactive) policy must be stochastic (i.e. randomized).

In 2-player games like Rock-Paper-Scissors, a random strategy is also required in order to make agent choices unpredictable to the opponent.





# Models of optimality

Is a fast nickel worth a slow dime?

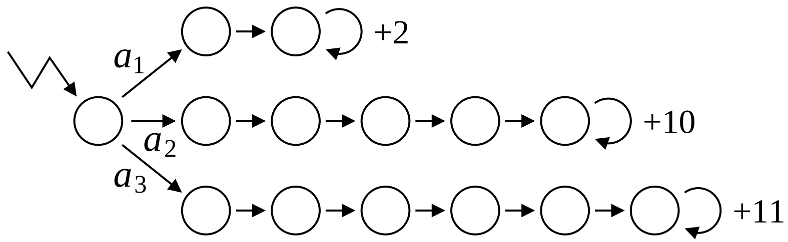
| | |
|---|---|
| Finite horizon reward | $\sum_{i=0}^{h-1} r_{t+i}$ |
| Infinite discounted reward | $\sum_{i=0}^{\infty} \gamma^i r_{t+i}, \qquad 0 \le \gamma < 1$ |
| Average reward | $\lim_{h\to\infty} \frac{1}{h} \sum_{i=0}^{h-1} r_{t+i}$ |

- Finite horizon reward is simple computationally
- Infinite discounted reward is easier for proving theorems
- Average reward is hard to deal with, because can't sensibly choose between small reward soon and large reward very far in the future.





# Comparing Models of Optimality




- Finite horizon, $k = 4$ $\rightarrow$ $a_1$ is preferred
- Infinite horizon, $\gamma = 0.9$ $\rightarrow$ $a_2$ is preferred
- Average reward $\rightarrow$ $a_3$ is preferred

# RL Approaches

- Value Function Learning
  - TD-Learning
  - Q-Learning
- Policy Learning
  - Hill Climbing
  - Policy Gradients
  - Evolutionary Strategy
- Actor-Critic
  - combination of Value and Policy learning





# Value Function Learning

Every policy $\pi$ determines a Value Function $V^\pi : \mathcal{S} \to \mathrm{R}$ where $V^\pi(s)$ is the average discounted reward received by an agent who begins in state $s$ and chooses its actions according to policy $\pi$.

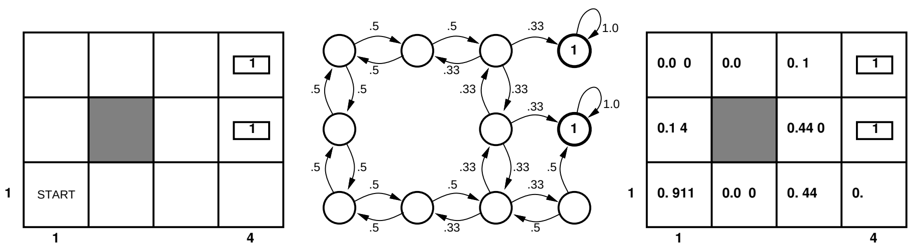
If $\pi = \pi^*$ is optimal, then $V^*(s) = V^{\pi^*}(s)$ is the maximum (expected) discounted reward obtainable from state $s$. Learning this optimal value function can help to determine the optimal strategy.

The agent retains its own estimate $V()$ of the "true" value function $V^*()$. The aim of Value Function Learning is generally to start with a random $V$ and then iteratively improve it so that it more closely approximates $V^*$. This process is sometimes called "Bootstrapping".





# Value Function




(a) (b) (c)

This is the Value Function $V^\pi$ where $\pi$ is the policy of choosing between available actions uniformly randomly.





# K-Armed Bandit Problem


The special case of an active, stochastic environment with only one state is called the K-armed Bandit Problem, because it is like being in a room with several (friendly) slot machines, for a limited time, and trying to collect as much money as possible.
Each action (slot machine) provides a different average reward.

# Exploration / Exploitation Tradeoff

Most of the time we should choose what we think is the best action.

However, in order to ensure convergence to the optimal strategy, we must occasionally choose something different from our preferred action, e.g.

- choose a random action 5% of the time, or
- use Softmax (Boltzmann distribution) to choose the next action:

$$P(a) = \frac{e^{\mathcal{R}(a))/T}}{\sum_{b \in \mathcal{A}} e^{\mathcal{R}(b))/T}}$$





# Exploration / Exploitation Tradeoff

I was born to try...

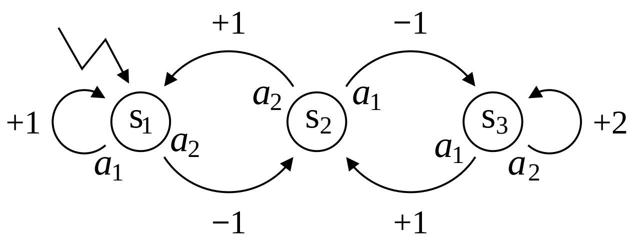
But you've got to make choices
Be wrong or right
Sometimes you've got to sacrifice the things you like.

- Delta Goodrem





# Delayed Reinforcement




We may need to take several actions before we can get the good reward.





# Temporal Difference Learning

Let's first assume that $\mathcal{R}$ and $\delta$ are deterministic. Then the (true) value $V^*(s)$ of the current state $s$ should be equal to the immediate reward plus the discounted value of the next state

$$V^*(s) = \mathcal{R}(s,a) + \gamma V^*(\delta(s,a))$$

We can turn this into an update rule for the estimated value, i.e.

$$V(s_t) \leftarrow r_t + \gamma V(s_{t+1})$$

If $\mathcal{R}$ and $\delta$ are stochastic (multi-valued), it is not safe to simply replace $V(s)$ with the expression on the right hand side. Instead, we move its value fractionally in this direction, proportional to a learning rate $\eta$

$$V(s_t) \leftarrow V(s_t) + \eta\,[r_t + \gamma V(s_{t+1}) - V(s_t)]$$

# Q-Learning

Q-Learning is similar to TD-Learning except that we use a function $Q^\pi : \mathcal{S} \times \mathcal{A} \to \mathrm{R}$ which depends on a state, action pair instead of just a state.

For any policy $\pi$ the Q-Function $Q^\pi(s,a)$ is the average discounted reward received by an agent who begins in state $s$, first performs action $a$ and then follows policy $\pi$ for all subsequent timesteps.

If $\pi = \pi^*$ is optimal, then $Q^*(s,a) = Q^{\pi^*}(s,a)$ is the maximum (expected) discounted reward obtainable from $s$, if the agent is forced to take action $a$ in the first timestep but can act optimally thereafter.

The agent retains its own (initially, random) estimate $Q()$ and iteratively improves this estimate to more closely approximate the "true" function $Q^*()$.





# Q-Learning

For a deterministic environment, $\pi^*$, $Q^*$ and $V^*$ are related by

$$\pi^*(s) = \mathrm{argmax}_a\, Q^*(s,a)$$
$$Q^*(s,a) = \mathcal{R}(s,a) + \gamma V^*(\delta(s,a))$$
$$V^*(s) = \max_b Q^*(s,b)$$

So

$$Q^*(s,a) = \mathcal{R}(s,a) + \gamma \max_b Q^*(\delta(s,a),b)$$

This allows us to iteratively approximate $Q$ by

$$Q(s_t,a_t) \leftarrow r_t + \gamma \max_b Q(s_{t+1},b)$$

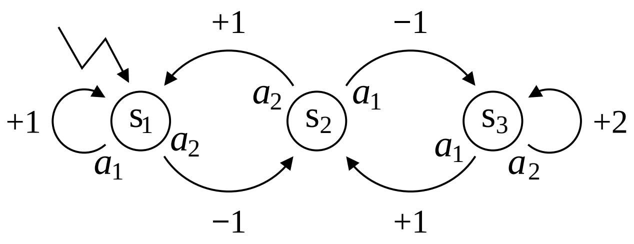
If the environment is stochastic, we instead write

$$Q(s_t,a_t) \leftarrow Q(s_t,a_t) + \eta\,[r_t + \gamma \max_b Q(s_{t+1},b) - Q(s_t,a_t)]$$





# Q-Learning Example




Exercise:

1. compute $V^\pi(s_3)$ if $\pi(s_3) = a_2$ and $\gamma = 0.9$
2. compute $\pi^*$, $V^*$ and $Q^*$ for this environment (if $\gamma = 0.9$)





# Theoretical Results

Theorem: Q-learning will eventually converge to the optimal policy, for any deterministic Markov decision process, assuming an appropriately randomized strategy.

(Watkins & Dayan 1992)

Theorem: TD-learning will also converge, with probability 1.

(Sutton 1988, Dayan 1992, Dayan & Sejnowski 1994)

# Limitations of Theoretical Results

- Delayed reinforcement
  - reward resulting from an action may not be received until several time steps later, which also slows down the learning
- Search space must be finite
  - convergence is slow if the search space is large
  - relies on visiting every state infinitely often
- For “real world” problems, we can’t rely on a lookup table
  - need to have some kind of generalisation (e.g. TD-Gammon)





# Computer Game Playing

Suppose we want a write a computer program to play a game like Backgammon, Chess, Checkers or Go. This can be done using a tree search algorithm (expectimax, MCTS, or minimax with alpha-beta pruning). But we need:

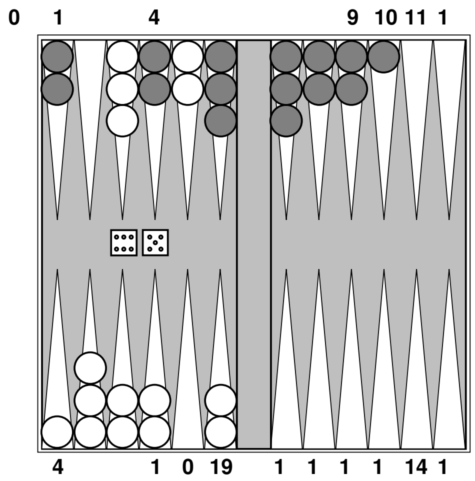
(a) an appropriate way of encoding any board position as a set of numbers, and

(b) a way to train a neural network or other learning system to compute a board evaluation, based on those numbers





# Backgammon








# Backgammon Neural Network

**Board encoding**

- 4 units × 2 players × 24 points
- 2 units for the bar
- 2 units for off the board

**Two layer neural network**

- 196 input units
- 20 hidden units
- 1 output unit

The input $s$ is the encoded board position (state),
the output $V(s)$ is the value of this position (probability of winning).

At each move, roll the dice, find all possible “next board positions”, convert them to the appropriate input format, feed them to the network, and choose the one which produces the largest output.

# Backpropagation

$$w \leftarrow w + \eta(T - V)\frac{\partial V}{\partial w}$$

$$\begin{aligned} V &= \text{actual output} \\ T &= \text{target value} \\ w &= \text{weight} \\ \eta &= \text{learning rate} \end{aligned}$$

Q: How do we choose the target value $T$ ?

In other words, how do we know what the value of the current position "should have been"? or, how do we find a better estimate for the value of the current position?





# How to Choose the Target Value

- Behavioral Cloning (Supervised Learning)
  - learn moves from human games (Expert Preferences)
- Temporal Difference Learning
  - use subsequent positions to refine evaluation of current position
  - general method, does not rely on knowing the "world model" (rules of the game)
- methods which combine learning with tree search (must know the "world model")
  - TD-Root, TD-Leaf, MCTS, TreeStrap





# TD-Learning for Episodic Games

Backgammon is an example of an episodic task, in the sense that the agent receives just a single reward at the end of the game, which we can consider as the final value $V_{m+1}$ (typically, $+1$ for a win or $-1$ for a loss). We then have a sequence of game positions, each with its own (estimated) value:

$$\text{(current estimate)} \quad V_t \to V_{t+1} \to \ldots \to V_m \to V_{m+1} \quad \text{(final result)}$$

In this context, TD-Learning simplifies and becomes equivalent to using the value of the next state ($V_{t+1}$) as the training value for the current state ($V_k$)

A fancier version, called TD($\lambda$), uses $T_k$ as the training value for $V_k$, where

$$T_t = (1-\lambda)\sum_{k=t+1}^{m} \lambda^{k-1-t} V_k + \lambda^{m-t} V_{m+1}$$

$T_t$ is a weighted average of future estimates, $\lambda =$ discount factor ($0 \le \lambda < 1$)





# TD-Gammon

- Tesauro trained two networks:
  - EP-network was trained on Expert Preferences (Supervised)
  - TD-network was trained by self play (TD-Learning)
- TD-network outperformed the EP-network.
- With modifications such as 3-step lookahead (expectimax) and additional hand-crafted input features, TD-Gammon became the best Backgammon player in the world (Tesauro, 1995).

# Policy Learning

There is another class of Reinforcement Learning algorithms which do not optimize a Value function but instead try to optimize the Policy itself, directly.

Normally, we consider a family of policies $\pi_\theta : \mathcal{S} \to \mathcal{A}$ determined by parameters $\theta$ (for example, the weights of a neural network).

For episodic domains like Backgammon, we do not need a discount factor, and the "fitness" of policy $\pi_\theta$ can be taken as the Value function of the initial state $s_0$ under this policy, which is the expected (or average) total reward received in each game by an agent using policy $\pi_\theta$

$$\text{fitness}(\pi_\theta) = V^{\pi_\theta}(s_0) = \mathbf{E}_{\pi_\theta}(r_{\text{total}})$$





# Hill Climbing (Evolution Strategy)

- Initialize "champ" policy $\theta_{\text{champ}} = 0$
- for each trial, generate "mutant" policy

$$\theta_{\text{mutant}} = \theta_{\text{champ}} + \text{Gaussian noise (fixed } \sigma)$$

- champ and mutant are evaluated on the same task(s)
- if mutant does "better" than champ,

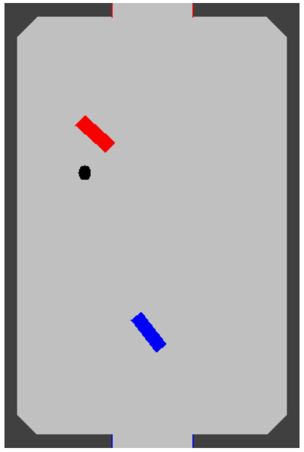
$$\theta_{\text{champ}} \leftarrow (1-\alpha)\theta_{\text{champ}} + \alpha\,\theta_{\text{mutant}}$$

- in some cases, the size of the update is scaled according to the difference in fitness (and may be negative)





# Case Study – Simulated Hockey



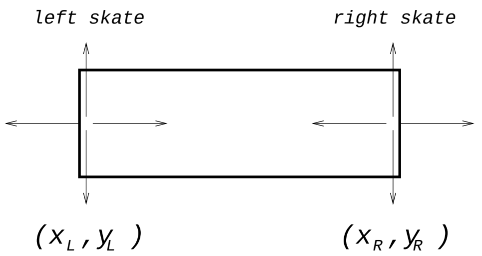




# Shock Physics

- rectangular rink with rounded corners
- near-frictionless playing surface
- "spring" method of collision handling
- frictionless puck (never acquires any spin)

## Shock Actuators

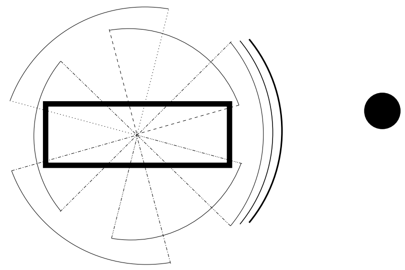



- a skate at each end of the vehicle with which it can push on the rink in two independent directions





## Shock Sensors



- 6 Braitenberg-style sensors equally spaced around the vehicle
- each sensor has an angular range of 90° with an overlap of 30° between neighbouring sensors





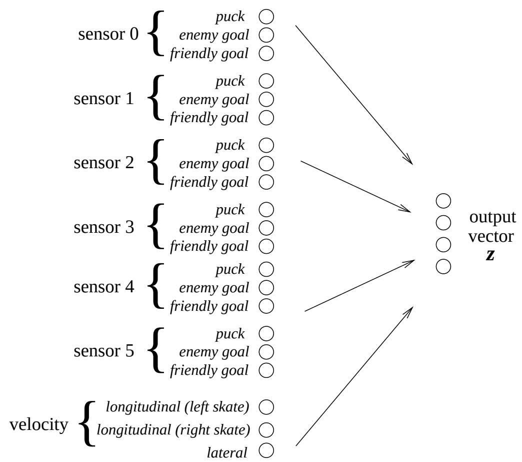
## Shock Inputs

- each of the 6 sensors responds to three different stimuli
  - ball / puck
  - own goal
  - opponent goal
- 3 additional inputs specify the current velocity of the vehicle
- total of $3 \times 6 + 3 = 21$ inputs





## Shock Agent



# Shock Agent

- single layer network with 21 inputs and 4 outputs
- total of $4 \times (21+1) = 88$ weights
- our "genome" (for Evolutionary Computation) consists of a vector of these 88 parameters
- mutation = add Gaussian random noise to each parameter, with standard deviation 0.05





# Shock Task

- each game begins with a random "game initial condition"
  - random position for puck
  - random position and orientation for player
- each game ends with
  - +1 if puck $\rightarrow$ enemy goal
  - -1 if puck $\rightarrow$ own goal
  - 0 if time limit expires





# Evolution Strategy

- mutant $\leftarrow$ champ + Gaussian noise
- champ and mutant play up to $n$ games, with same game initial conditions
- if mutant does "better" than champ,

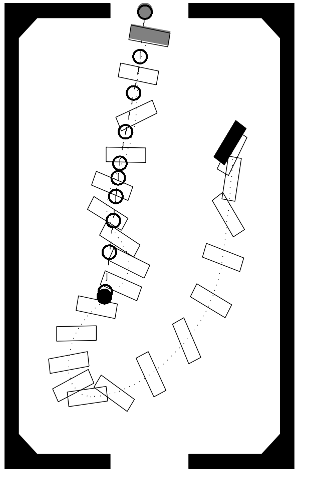
$$\text{champ} \leftarrow (1-\alpha)*\text{champ} + \alpha*\text{mutant}$$

- "better" means the mutant must score higher than the champ in the first game, and at least as high as the champ in each subsequent game





# Evolved Behavior



# HC-Gammon

- HC-Gammon was trained to play Backgammon using this Evolution Strategy
- same "game initial conditions" = same seed for generating dice rolls
- weights were used to determine value function, but the learning optimizes performance of policy directly rather than aiming to make value function more accurate
- performance was almost as good as TD-Gammon, but not quite
  - gradient information provides more precise updates, particularly for rarely used weights in the network






# Policy Gradients

Policy Gradients are an alternative to Evolution Strategy, which use gradient ascent rather than random updates.

Let's first consider episodic games. The agent takes a sequence of actions

$$a_1\ a_2\ \dots\ a_t\ \dots\ a_m$$

At the end it receives a reward $r_{\text{total}}$. We don't know which actions contributed the most, so we just reward all of them equally. If $r_{\text{total}}$ is high (low), we change the parameters to make the agent more (less) likely to take the same actions in the same situations. In other words, we want to increase (decrease)

$$\log \prod_{t=1}^{m} \pi_\theta(a_t|s_t) = \sum_{t=1}^{m} \log \pi_\theta(a_t|s_t)$$






# Policy Gradients

If $r_{\text{total}} = +1$ for a win and $-1$ for a loss, we can simply multiply the log probability by $r_{\text{total}}$. Differentials can be calculated using the gradient

$$\nabla_\theta\, r_{\text{total}} \sum_{t=1}^{m} \log \pi_\theta(a_t|s_t) = r_{\text{total}} \sum_{t=1}^{m} \nabla_\theta \log \pi_\theta(a_t|s_t)$$

The gradient of the log probability can be calculated nicely using Softmax.

If $r_{\text{total}}$ takes some other range of values, we can replace it with $(r_{\text{total}} - b)$ where $b$ is a fixed value, called the baseline.






# REINFORCE Algorithm

We then get the following REINFORCE algorithm:

for each trial
  run trial and collect states $s_t$, actions $a_t$, and reward $r_{\text{total}}$
  for $t$ = 1 to length(trial)
    $\theta \leftarrow \theta + \eta(r_{\text{total}} - b)\nabla_\theta \log \pi_\theta(a_t|s_t)$
  end
end

This algorithm has successfully been applied, for example, to learn to play the game of Pong from raw image pixels.

# Policy Gradients

We wish to extend the framework of Policy Gradients to non-episodic domains, where rewards are received incrementally throughout the game (e.g. PacMan, Space Invaders).

Every policy $\pi_\theta$ determines a distribution $\rho_{\pi_\theta}(s)$ on $\mathcal{S}$

$$\rho_{\pi_\theta}(s) = \sum_{t\geq 0} \gamma^t \mathrm{prob}_{\pi_\theta,t}(s)$$

where $\mathrm{prob}_{\pi_\theta,t}(s)$ is the probability that, after starting in state $s_0$ and performing $t$ actions, the agent will be in state $s$. We can then define the fitness of policy $\pi$ as

$$\mathrm{fitness}(\pi_\theta) = \sum_s \rho_{\pi_\theta}(s) \sum_a Q^{\pi_\theta}(s,a)\, \pi_\theta(a|s)$$






# Policy Gradients

$$\mathrm{fitness}(\pi_\theta) = \sum_s \rho_{\pi_\theta}(s) \sum_a Q^{\pi_\theta}(s,a)\, \pi_\theta(a|s)$$

Note: In the case of episodic games, we can take $\gamma = 1$, in which case $Q^{\pi_\theta}(s,a)$ is simply the expected reward at the end of the game. However, the above equation holds in the non-episodic case as well.

The gradient of $\rho_{\pi_\theta}(s)$ and $Q^{\pi_\theta}(s,a)$ are extremely hard to determine, so we ignore them and instead compute the gradient only for the last term $\pi_\theta(a|s)$.

$$\nabla_\theta\, \mathrm{fitness}(\pi_\theta) = \sum_s \rho_{\pi_\theta}(s) \sum_a Q^{\pi_\theta}(s,a) \nabla_\theta\, \pi_\theta(a|s)$$






# The Log Trick

$$\begin{aligned}\sum_a Q^{\pi_\theta}(s,a)\, \nabla_\theta\, \pi_\theta(a|s) &= \sum_a Q^{\pi_\theta}(s,a)\, \pi_\theta(a|s)\, \frac{\nabla_\theta\, \pi_\theta(a|s)}{\pi_\theta(a|s)} \\ &= \sum_a Q^{\pi_\theta}(s,a)\, \pi_\theta(a|s)\, \nabla_\theta \log \pi_\theta(a|s)\end{aligned}$$

So

$$\begin{aligned}\nabla_\theta\, \mathrm{fitness}(\pi_\theta) &= \sum_s \rho_{\pi_\theta}(s) \sum_a Q^{\pi_\theta}(s,a)\, \pi_\theta(a|s)\, \nabla_\theta \log \pi_\theta(a|s) \\ &= \mathbf{E}_{\pi_\theta}[\, Q^{\pi_\theta}(s,a) \nabla_\theta \log \pi_\theta(a|s)\,]\end{aligned}$$

The reason for the last equality is this:

$\rho_{\pi_\theta}(s)$ is the number of times (discounted by $\gamma^t$) that we expect to visit state $s$ when using policy $\pi_\theta$. Whenever state $s$ is visited, action $a$ will be chosen with probability $\pi_\theta(a|s)$.






# Actor-Critic

Recall:

$$\nabla_\theta\, \mathrm{fitness}(\pi_\theta) = \mathbf{E}_{\pi_\theta}[\, Q^{\pi_\theta}(s,a) \nabla_\theta \log \pi_\theta(a|s)\,]$$

For non-episodic games, we cannot easily find a good estimate for $Q^{\pi_\theta}(s,a)$. One approach is to consider a family of Q-Functions $Q_w$ determined by parameters $w$ (different from $\theta$) and learn $w$ so that $Q_w$ approximates $Q^{\pi_\theta}$, at the same time that the policy $\pi_\theta$ itself is also being learned.

This is known as an Actor-Critic approach because the policy determines the action, while the Q-Function estimates how good the current policy is, and thereby plays the role of a critic.

# Actor Critic Algorithm

for each trial

  sample $a_0$ from $\pi(a|s_0)$

  for each timestep $t$ do

    sample reward $r_t$ from $\mathcal{R}(r\,|\,s_t, a_t)$

    sample next state $s_{t+1}$ from $\delta(s\,|\,s_t, a_t)$

    sample action $a_{t+1}$ from $\pi(a\,|\,s_{t+1})$

    $\frac{dE}{dQ} = -[r_t + \gamma Q_w(s_{t+1}, a_{t+1}) - Q_w(s_t, a_t)]$

    $\theta \leftarrow \theta + \eta_\theta\, Q_w(s_t, a_t) \nabla_\theta \log \pi_\theta(a_t\,|\,s_t)$

    $w \leftarrow w - \eta_w \frac{dE}{dQ} \nabla_w\, Q_w(s_t, a_t)$

  end

end