13. Autoencoders

Textbook, Chapter 14
Outline

- Autoencoder Networks (14.1)
- Regularized Autoencoders (14.2)
- Stochastic Encoders and Decoders (14.4)
- Generative Models
- Variational Autoencoders (20.10.3)
Recall: Encoder Networks

- Identity mapping through a bottleneck
- Also called N–M–N task
- Used to investigate hidden unit representations

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Autoencoder Networks

- output is trained to reproduce the input as closely as possible
- activations normally pass through a bottleneck, so the network is forced to compress the data in some way
- like the RBM, Autoencoders can be used to automatically extract abstract features from the input
Autoencoder Networks

If the encoder computes $z = f(x)$ and the decoder computes $g(f(x))$ then we aim to minimize some distance function between $x$ and $g(f(x))$

$$E = L(x, g(f(x)))$$
Autoencoder as Pretraining

- after an autoencoder is trained, the decoder part can be removed and replaced with, for example, a classification layer
- this new network can then be trained by backpropagation
- the features learned by the autoencoder then serve as initial weights for the supervised learning task
Greedy Layerwise Pretraining

- Autoencoders can be used as an alternative to Restricted Boltzmann Machines, for greedy layerwise pretraining.
- An autoencoder with one hidden layer is trained to reconstruct the inputs. The first layer (encoder) of this network becomes the first layer of the deep network.
- Each subsequent layer is then trained to reconstruct the previous layer.
- A final classification layer is then added to the resulting deep network, and the whole thing is trained by backpropagation.
Avoiding Trivial Identity

- if there are more hidden nodes than inputs (which often happens in image processing) there is a risk the network may learn a trivial identity mapping from input to output
- we generally to avoid this by introducing some form of regularization
Regularized Autoencoders (14.2)

- sparse autoencoders
- autoencoders with dropout at hidden layer(s)
- contractive autoencoders
- denoising autoencoders
Sparse Autoencoder (14.2.1)

- One way to regularize an autoencoder is to add a penalty term to the cost function, based on the hidden unit activations.

- This is analogous to the weight decay term we previously used for supervised learning.

- One popular choice is to penalize the sum of the absolute values of the activations in the hidden layer:

\[ E = L(x, g(f(x))) + \lambda \sum_i |h_i| \]

- This is sometimes known as L₁-regularization (because it involves the absolute value rather than the square); it can encourage some of the hidden units to go to zero, thus producing a sparse representation.
Contractive Autoencoder (14.2.3)

- another popular penalty term is the $L_2$-norm of the derivatives of the hidden units with respect to the inputs

$$E = L(x, g(f(x)) + \lambda \sum_i ||\nabla_x h_i||^2$$

- this forces the model to learn hidden features that do not change much when the training inputs $x$ are slightly altered
Denoising Autoencoder (14.2.2)

Another regularization method, similar to contractive autoencoder, is to add noise to the inputs, but train the network to recover the original input.

repeat:
    sample a training item $x^{(i)}$
    generate a corrupted version $\tilde{x}$ of $x^{(i)}$
    train to reduce $E = L(x^{(i)}, g(f(\tilde{x})))$
end
Cost Functions and Probability

- We saw previously how the loss (cost) function at the output of a feedforward neural network (with parameters $\theta$) can be seen as defining a probability distribution $p_{\theta}(x)$ over the outputs. We then train to maximize the log of the probability of the target values.
  
  - squared error assumes an underlying Gaussian distribution, whose mean is the output of the network
  - cross entropy assumes a Bernoulli distribution, with probability equal to the output of the network
  - softmax assumes a Boltzmann distribution
Stochastic Encoders and Decoders (14.4)

- For autoencoders, the decoder can be seen as defining a conditional probability distribution $p_\theta(x|z)$ of output $x$ for a certain value $z$ of the hidden or “latent” variables.

- In some cases, the encoder can also be seen as defining a conditional probability distribution $q_\phi(z|x)$ of latent variables $z$ based on an input $x$.

- We have seen an example of this with the Restricted Boltzmann Machine, where $q_\phi(z|x)$ and $p_\theta(x|z)$ were Bernoulli distributions.
Generative Models

- Sometimes, as well as reproducing the training items $\{x^{(i)}\}$, we also want to be able to use the decoder to generate new items which are of a similar “style” to the training items.

- In other words, we want to be able to choose latent variables $z$ from a standard Normal distribution $p(z)$, feed these values of $z$ to the decoder, and have it produce a new item $x$ which is somehow similar to the training items.

- Generative models can be:
  - explicit (Variational Autoencoders)
  - implicit (Generative Adversarial Networks)
Variational Autoencoder (20.10.3)

Instead of producing a single $z$ for each $x^{(i)}$, the encoder (with parameters $\phi$) can be made to produce a mean $\mu_{z|x^{(i)}}$ and standard deviation $\Sigma_{z|x^{(i)}}$. This defines a conditional (Normal) probability distribution $q_\phi(z|x^{(i)})$. We then train the system to maximize

$$\mathbb{E}_{z \sim q_\phi(z|x^{(i)})} \left[ \log p_\theta(x^{(i)}|z) \right] - D_{KL}(q_\phi(z|x^{(i)}) \parallel p(z))$$

- the first term enforces that any sample $z$ drawn from the conditional distribution $q_\phi(z|x^{(i)})$ should, when fed to the decoder, produce something approximating $x^{(i)}$
- the second term encourages $q_\phi(z|x^{(i)})$ to approximate $p(z)$
- in practice, the distributions $q_\phi(z|x^{(i)})$ for various $x^{(i)}$ will occupy complementary regions within the overall distribution $p(z)$
Variational Autoencoder Digits
Variational Autoencoder Digits

1st Epoch

9th Epoch

Original
Variational Autoencoder Faces
Variational Autoencoder

- Variational Autoencoder produces reasonable results
- tends to produce blurry images
- often end up using only a small number of the dimensions available to $z$

References:

http://kvfrans.com/variational-autoencoders-explained/