Hill Climbing (Evolution Strategy)

- Initialize “champ” policy $\theta_{champ} = 0$
- for each trial, generate “mutant” policy
  $$\theta_{mutant} = \theta_{champ} + \text{Gaussian noise (fixed } \sigma)$$
- champ and mutant are evaluated on the same task(s)
- if mutant does “better” than champ,
  $$\theta_{champ} \leftarrow (1 - \alpha)\theta_{champ} + \alpha \theta_{mutant}$$
- in some cases, the size of the update is scaled according to the difference in fitness (and may be negative)
Shock Sensors

- 6 Braitenberg-style sensors equally spaced around the vehicle
- each sensor has an angular range of 90° with an overlap of 30° between neighbouring sensors

Shock Inputs

- each of the 6 sensors responds to three different stimuli
  - ball / puck
  - own goal
  - opponent goal
- 3 additional inputs specify the current velocity of the vehicle
- total of $3 \times 6 + 3 = 21$ inputs

Policy Gradients

Policy Gradients are an alternative to Evolution Strategy, which use gradient ascent rather than random updates.

Let’s first consider episodic games. The agent takes a sequence of actions

$$a_1 a_2 \ldots a_t \ldots a_m$$

At the end it receives a reward $r_{\text{total}}$. We don’t know which actions contributed the most, so we just reward all of them equally. If $r_{\text{total}}$ is high (low), we change the parameters to make the agent more (less) likely to take the same actions in the same situations. In other words, we want to increase (decrease)

$$\log \prod_{t=1}^m \pi_0(a_t | s_t) = \sum_{t=1}^m \log \pi_0(a_t | s_t)$$
Policy Gradients

We wish to extend the framework of Policy Gradients to non-episodic domains, where rewards are received incrementally throughout the game (e.g. PacMan, Space Invaders). Every policy $\pi^\theta$ determines a distribution $\rho^\pi_\theta(s)$ on $S$:

$$\rho^\pi_\theta(s) = \sum_{t \geq 0} \gamma^t \text{prob}^\pi_\theta(s)$$

where $\text{prob}^\pi_\theta(s)$ is the probability that, after starting in state $s_0$ and performing $t$ actions, the agent will be in state $s$. We can then define the fitness of policy $\pi$ as:

$$\text{fitness}(\pi^\theta) = \sum_s \rho^\pi_\theta(s) \sum_a Q^\pi_\theta(s,a) \pi^\theta(a|s)$$

Note: In the case of episodic games, we can take $\gamma = 1$, in which case $Q^\pi_\theta(s,a)$ is simply the expected reward at the end of the game. However, the above equation holds in the non-episodic case as well. The gradient of $\rho^\pi_\theta(s)$ and $Q^\pi_\theta(s,a)$ are extremely hard to determine, so we ignore them and instead compute the gradient only for the last term $\pi^\theta(a|s)$.

$$\nabla_\theta \text{fitness}(\pi^\theta) = \sum_s \rho^\pi_\theta(s) \sum_a Q^\pi_\theta(s,a) \nabla_\theta \pi^\theta(a|s)$$

REINFORCE Algorithm

We then get the following REINFORCE algorithm:

for each trial
  run trial and collect states $s_t$, actions $a_t$, and reward $r_{total}$
  for $t = 1$ to length(trial)
    $\theta \leftarrow \theta + \eta(r_{total} - b) \nabla_\theta \log \pi^\theta(a_t|s_t)$
  end
end

This algorithm has successfully been applied, for example, to learn to play the game of Pong from raw image pixels.

If $r_{total} = +1$ for a win and $-1$ for a loss, we can simply multiply the log probability by $r_{total}$. Differentials can be calculated using the gradient

$$\nabla_\theta r_{total} = r_{total} \sum_{t=1}^{m} \nabla_\theta \log \pi^\theta(a_t|s_t)$$

The gradient of the log probability can be calculated nicely using Softmax. If $r_{total}$ takes some other range of values, we can replace it with $(r_{total} - b)$ where $b$ is a fixed value, called the baseline.
## The Log Trick

\[
\sum_a Q^\pi_\theta(s,a) \nabla_\theta \pi_\theta(a|s) = \sum_a Q^{\pi_\theta}(s,a) \pi_\theta(a|s) \frac{\nabla_\theta \pi_\theta(a|s)}{\pi_\theta(a|s)} = \sum_a Q^{\pi_\theta}(s,a) \pi_\theta(a|s) \nabla_\theta \log \pi_\theta(a|s)
\]

So

\[
\nabla_\theta \text{fitness}(\pi_\theta) = \sum_s \rho_\pi_\theta(s) \sum_a Q^{\pi_\theta}(s,a) \pi_\theta(a|s) \nabla_\theta \log \pi_\theta(a|s)
\]

\[
= E_{\pi_\theta} [Q^{\pi_\theta}(s,a) \nabla_\theta \log \pi_\theta(a|s)]
\]

The reason for the last equality is this:

\[
\rho_\pi_\theta(s) \text{ is the number of times (discounted by } \gamma^t) \text{ that we expect to visit state } s \text{ when using policy } \pi_\theta. \text{ Whenever state } s \text{ is visited, action } a \text{ will be chosen with probability } \pi_\theta(a|s).
\]

## Actor-Critic

Recall:

\[
\nabla_\theta \text{fitness}(\pi_\theta) = E_{\pi_\theta} [Q^{\pi_\theta}(s,a) \nabla_\theta \log \pi_\theta(a|s)]
\]

For non-episodic games, we cannot easily find a good estimate for \(Q^{\pi_\theta}(s,a)\). One approach is to consider a family of Q-Functions \(Q_w\), determined by parameters \(w\) (different from \(\theta\)) and learn \(w\) so that \(Q_w\) approximates \(Q^{\pi_\theta}\), at the same time that the policy \(\pi_\theta\) itself is also being learned.

This is known as an Actor-Critic approach because the policy determines the action, while the Q-Function estimates how good the current policy is, and thereby plays the role of a critic.

## Actor Critic Algorithm

for each trial
  sample \(a_0\) from \(\pi(a|s_0)\)

for each timestep \(t\) do
  sample reward \(r_t\) from \(R(r|s_t,a_t)\)
  sample next state \(s_{t+1}\) from \(\delta(s|s_t,a_t)\)
  sample action \(a_{t+1}\) from \(\pi(a|s_{t+1})\)

  \[
  \frac{dE}{dQ} = -[r_t + \gamma Q_w(s_{t+1},a_{t+1}) - Q_w(s_t,a_t)]
  \]

  \[
  \theta \leftarrow \theta + \eta_\theta Q_w(s_t,a_t) \nabla_\theta \log \pi_\theta(a_t|s_t)
  \]

  \[
  w \leftarrow w - \eta_w \frac{dE}{dQ} \nabla_w Q_w(s_t,a_t)
  \]

end

## Reinforcement Learning Timeline

- **model-free methods**
  - 1961 MENACE tic-tac-toe (Donald Michie)
  - 1986 TD(\(\lambda\)) (Rich Sutton)
  - 1989 TD-Gammon (Gerald Tesauro)
  - 2015 Deep Q Learning for Atari Games
  - 2016 A3C (Mnih et al.)
  - 2017 OpenAI Evolution Strategies (Salimans et al.)

- **methods relying on a world model**
  - 1959 Checkers (Arthur Samuel)
  - 1997 TD-leaf (Baxter et al.)
  - 2009 TreeStrap (Veness et al.)
  - 2016 Alpha Go (Silver et al.)
MENACE

Machine Educable Noughts And Crosses Engine
Donald Michie, 1961

Game Tree (2-player, deterministic)

Terminology

MAX (X)  MIN (O)

MIN (O)

MAX (X)

MIN (O)

TERMINAL

Utility

-1  0  +1

Martin Gardner and HALO

The Unexpected Hanging and Other Mathematical Diversions
Martin Gardner

Alan Blair, 2017-20
Hexapawn Boxes

Deep Q-Learning for Atari Games

- end-to-end learning of values $Q(s, a)$ from pixels $s$
- input state $s$ is stack of raw pixels from last 4 frames
  - 8-bit RGB images, $210 \times 160$ pixels
- output is $Q(s, a)$ for 18 joystick/button positions
- reward is change in score for that timestep

Reinforcement Learning with BOXES

- this BOXES algorithm was later adapted to learn more general tasks such as Pole Balancing, and helped lay the foundation for the modern field of Reinforcement Learning
- for various reasons, interest in Reinforcement Learning faded in the late 70’s and early 80’s, but was revived in the late 1980’s, largely through the work of Richard Sutton
- Gerald Tesauro applied Sutton’s TD-Learning algorithm to the game of Backgammon in 1989

Deep Q-Network
Q-Learning

\[ Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \eta [r_t + \gamma \max_b Q(s_{t+1}, b) - Q(s_t, a_t)] \]

- with lookup table, Q-learning is guaranteed to eventually converge
- for serious tasks, there are too many states for a lookup table
- instead, \( Q_w(s, a) \) is parametrized by weights \( w \), which get updated so as to minimize
  \[ [r_t + \gamma \max_b Q_w(s_{t+1}, b) - Q_w(s_t, a_t)]^2 \]
  - note: gradient is applied only to \( Q_w(s_t, a_t) \), not to \( Q_w(s_{t+1}, b) \)
- this works well for some tasks, but is challenging for Atari games, partly due to temporal correlations between samples (i.e. large number of similar situations occurring one after the other)

Deep Q-Learning with Experience Replay

- choose actions using current Q function (\( \varepsilon \)-greedy)
- build a database of experiences \( (s_t, a_t, r_t, s_{t+1}) \)
- sample asynchronously from database and apply update, to minimize
  \[ [r_t + \gamma \max_b Q_w(s_{t+1}, b) - Q_w(s_t, a_t)]^2 \]
- removes temporal correlations by sampling from variety of game situations in random order
- makes it easier to parallelize the algorithm on multiple GPUs

DQN Results for Atari Games

Deep Q-Networks with Experience Replay

- Prioritised Replay
  - weight experience according to surprise
- Double Q-Learning
  - current Q-network \( w \) is used to select actions
  - older Q-network \( \overline{w} \) is used to evaluate actions
- Advantage Function
  - action-independent value function \( V_w(s) \)
  - action-dependent advantage function \( A_w(s, a) \)
  \[ Q(s, a) = V_w(s) + A_w(s, a) \]
Prioritised Replay

- instead of sampling experiences uniformly, store them in a priority queue according to the DQN error
  \[ |r_t + \gamma \max_b Q_w(s_{t+1}, b) - Q_w(s_t, a_t)| \]
- this ensures the system will concentrate more effort on situations where the Q value was “surprising” (in the sense of being far away from what was predicted)

Advantage Function

The Q Function \(Q^\pi(s, a)\) can be written as a sum of the value function \(V^\pi(s)\) plus an advantage function \(A^\pi(s, a) = Q^\pi(s, a) - V^\pi(s)\)

\(A^\pi(s, a)\) represents the advantage (or disadvantage) of taking action \(a\) in state \(s\), compared to taking the action preferred by the current policy \(\pi\).

We can learn approximations for these two components separately:

\[Q(s, a) = V_\theta(s) + A_w(s, a)\]

Note that actions can be selected just using \(A_w(s, a)\), because

\[\argmax_b Q(s_{t+1}, b) = \argmax_b A_w(s_{t+1}, b)\]

Double Q-Learning

- if the same weights \(w\) are used to select actions and evaluate actions, this can lead to a kind of confirmation bias
- could maintain two sets of weights \(w\) and \(\overline{w}\), with one used for selection and the other for evaluation (then swap their roles)
- in the context of Deep Q-Learning, a simpler approach is to use the current “online” version of \(w\) for selection, and an older “target” version \(\overline{w}\) for evaluation; we therefore minimize

\[ |r_t + \gamma Q_{\overline{w}}(s_{t+1}, \arg\max_b Q_w(s_{t+1}, b)) - Q_w(s_t, a_t)|^2 \]

- a new version of \(\overline{w}\) is periodically calculated from the distributed values of \(w\), and this \(\overline{w}\) is broadcast to all processors.

Advantage Actor Critic

Recall that in the REINFORCE algorithm, a baseline \(b\) could be subtracted from \(r_{total}\)

\[ \theta \leftarrow \theta + \eta (r_{total} - b) \nabla_\theta \log \pi_\theta(a_t | s_t) \]

In the actor-critic framework, \(r_{total}\) is replaced by \(Q(s_t, a_t)\)

\[ \theta \leftarrow \theta + \eta \theta Q(s_t, a_t) \nabla_\theta \log \pi_\theta(a_t | s_t) \]

We can also subtract a baseline from \(Q(s_t, a_t)\). This baseline must be independent of the action \(a_t\), but it could be dependent on the state \(s_t\). A good choice of baseline is the value function \(V_\theta(s)\), in which case the Q function is replaced by the advantage function

\[A_w(s, a) = Q(s, a) - V_\theta(s)\]
Asynchronous Advantage Actor Critic

- use policy network to choose actions
- learn a parameterized Value function $V_u(s)$ by TD-Learning
- estimate Q-value by n-step sample
  $$Q(s_t, a_t) = r_{t+1} + \gamma r_{t+2} + \ldots + \gamma^{n-1} r_{t+n} + \gamma^n V_u(s_{t+n})$$
- update policy by
  $$\theta \leftarrow \theta + \eta \partial_{\theta} \left[ Q(s_t, a_t) - V_u(s_t) \right] \log \pi_\theta(a_t | s_t)$$
- update Value function my minimizing
  $$\left[ Q(s_t, a_t) - V_u(s_t) \right]^2$$

Latest Research in Deep RL

- augment A3C with unsupervised auxiliary tasks
- encourage exploration, increased entropy
- encourage actions for which the rewards are less predictable
- concentrate on state features from which the preceding action is more predictable
- transfer learning (between tasks)
- inverse reinforcement learning (infer rewards from policy)
- hierarchical RL
- multi-agent RL

References