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Topic Information

Assessment: Take home exam (weighting = $\frac{4}{13}$ of total course mark — *but must do well in all parts of course*).

- Distributed: 11/06/98
- Due: 4:30pm 19/06/98 at CSE office (or postmarked that day)

Questions based on what is *said* in lectures.

Reference text: Philip Straffin, *Game Theory and Strategy*, Volume 36 New Mathematical Library, The Mathematical Association of America, 1993. (ISBN: 0-883850-637-9)

Slides: `http://www.cse.unsw.edu.au/~morri/comp9514/`

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Course Outline

Lecture 1: Introduction to game theory; Two-person zero-sum games
– Dominance/saddle points

Lecture 2: Two-person zero-sum games (continued)
– Mixed Strategies; Game trees

Lecture 3: Two-person zero-sum games (conclusion)
– Utility; Games against nature
Two-person nonzero-sum games
– Nash Equilibria

Lecture 4: Two-person nonzero-sum games (conclusion)
– Prisoner's dilemma; Cooperation

(Exercises at end of each lecture.)

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Game Theory

Study of how players *should rationally* play games.

Study traditional games: tic-tac-toe, bridge, poker, . . .

Abstract from and generalise study of these traditional games.

Applications to: political candidates attempting to win election, company strategies, biological prosperity, . . .

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What is a Game?

(Straffin 1993) Situation in which we have:

1. At least *2 Players*
2. Players have a number of courses of action available to them (i.e., *strategies*)
3. Strategies determine *outcome* of game
4. Each outcome has a set of numerical *payoffs* — one to each player in the game

Aim: each player would like an outcome giving them the highest payoff possible.

Elements of conflict and coordination.

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What Game Theory is Not!

WARNING! Real-life games are enormously complex and difficult to model.

Aim: Model important features of actual game in hope that we can gain some insight.

WARNING! Real-life players are not always rational!

WARNING! Game theory does not always give unique way to play game.

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Two-Person Zero-Sum Games

Begin by concentrating on two players.

Each player has a payoff associated with each outcome.

If payoffs add to zero: **Zero-sum game.**

Pure conflict between players.

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Let's Play a Game

(Haywood 1954) Battle of the Bismark Sea

1943 Japanese occupying northern New Guinea, Allies south.

Japanese convoy to reinforce troops via two routes:

1. north — rain and bad visibility predicted
2. south — fair weather

Allies send fighter aircraft to damage convoy:

1. north
2. south

Payoff: number of days bombing available

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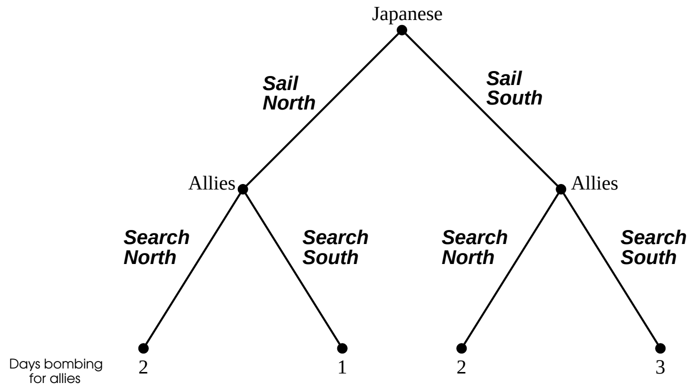
Game Tree

Extensive form of game.



What's good for Allies is going to be bad for Japanese army

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Two-Person Zero-Sum Games

Begin by concentrating on two player games.
Matrix games: Games in which payoffs associated with available strategies can be represented by and $n \times m$ matrix.

Each player has payoff associated with outcome.

		Player 2	
		Sail North	**Sail South**
Player 1	**Search North**	$(2,\ -2)$	$(2,\ -2)$
	Search South	$(1,\ -1)$	$(3,\ -3)$

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Payoffs add to zero. More compact representation:

		Player 2	
		Sail North	**Sail South**
Player 1	**Search North**	2	2
	Search South	1	3

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Payoff Matrix

Normal form of game.

CONVENTION: Entries represent payoff to row player

		Player 2	
		Sail North	**Sail South**
Player 1	**Search North**	2 days	2 days
	Search South	1 day	3 days

Minimax Strategy

Rational decision maker seeks action with best possible payoff in worst-case situation (best payoff assuming opponent makes best counter move).

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Let's Play a Game

(Straffin 1993)

		Player 2			
		A	**B**	**C**	**D**
Player 1	**A**	12	-1	1	0
	B	5	1	7	-20
	C	3	2	4	3
	D	-16	0	0	16

Compare with results on p. 7.

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Dominance Principle

Definition: A strategy S *dominates* a strategy T if:

1. every payoff in S is *at least* as good as corresponding payoff in T
2. at least one payoff in S is *strictly* better than corresponding payoff in T.

Idea: *Never* play a dominated strategy.

Dominance Principle: Rational player should never choose a dominated strategy.

Good start but does not recommend unique strategy in general.

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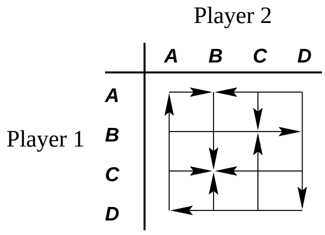
Equilibrium Outcome

Movement diagram. Draw:
row — arrow from each entry to smallest entry
column — arrow from each entry to largest entry



Equilibrium pair of strategies represent strategies where a player deciding to unilaterally deviate from this action will worsen their expected outcome.

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Saddle Point

Definition: An entry (outcome) is called a *saddle point* in a matrix game if it is less than or equal to any entry in its row and greater than or equal to any entry in its column.

Saddle Point Principle: If matrix game contains a saddle point both players should play strategy that contains it

Definition: In a matrix game, if there is a value v where the row player has a strategy guaranteeing at least v and the column player has a strategy guaranteeing row player no more than v, then v is the *value* of the game.

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Finding Saddle Points

Check each point (smallest in row and largest in column)

Alternatively:

1. determine minimum in each row — circle maximum of these
2. determine maximum in each column — circle minimum

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Player 1 (rows) vs. Player 2 (columns)

	A	B	C	D	*Row Min*	
A	12	-1	1	0	-1	
B	5	1	7	-20	-20	
C	3	2	4	3	2	⇐
D	-16	0	0	16	-16	
Col Max	12	2	7	16		
		⇑				

If circled entries coincide, they are saddle points. Otherwise they are not. There may be no saddle points in a game (we look at this situation next week).

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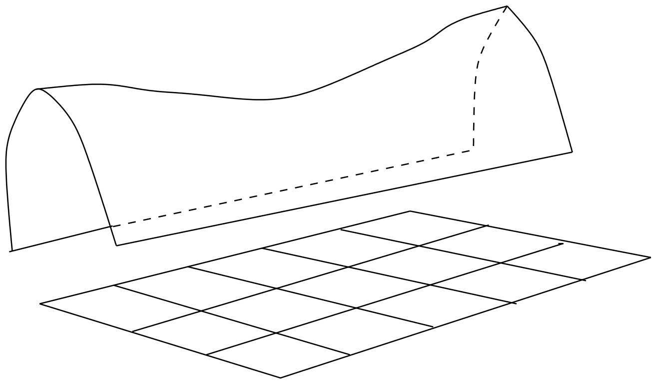
Why is it called a *Saddle Point*?



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Important Fact about Saddle Points

Theorem: Any two saddle points in a matrix game have the same value.

Moreover, if row and column player both play strategies with saddle points, the outcome is always a saddle point.

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Reflection on Saddle Points

Decision by two players that neither can *unilaterally* improve upon.

Either player could announce their choice of strategy beforehand and not be worse off!

Solution in *pure strategies.*

PURE STRATEGY — strategy says always to take the same action (otherwise mixed strategy).

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Why Study Two-Person Games?

Optimal strategy always exists (zero-sum games — Minimax theorem; nonzero-sum games — Nash's theorem)

Many situations with seemingly more "players" can be reduced to two-person games

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Terminology

ZERO-SUM GAME — payoffs add to zero

STRATEGY — course of action

PURE STRATEGY — strategy says to always take same action

MIXED STRATEGY — strategy varies with random factor

SOLUTION — strategy giving best possible payoff (a *regret-free* choice).

MULTISTAGE (ITERATED) GAME — game involving sequence of choices

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Exercise 1

(Drescher 1981)

Consider the following matrix game.

		Player 2		
		A	**B**	**C**
Player 1	**A**	1	-3	-2
	B	2	5	4
	C	2	3	2

Check for dominated strategies and saddle points.

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Exercise 2

(Straffin 1993)

Consider the following matrix game.

Player 2

Player 1		A	B	C	D
	A	4	2	5	2
	B	2	1	-1	-20
	C	3	2	4	2
	D	-16	10	16	1

Check for dominated strategies and saddle points.

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Exercise 3

Consider the game *Rock, Scissors, Paper.* Write the matrix for this game. Check for dominated strategies and saddle points.

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Exercise 4

(Williams 1954) The Coal Problem

Typically it takes 15 tons of coal to heat a house during winter but it can be as low as 10 tons or as high as 20 tons. The price of coal changes with the weather being \$10, \$15 and \$20 per ton during mild, normal and severe winters. You can buy now at \$10.

What should you do? Buy all or part of supply now?

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Exercise 5

(Williams 1954) The Secondhand Car

Two brothers inherit a car worth \$800. They decide to settle ownership by sealed bids. Bids are in hundred-dollar amounts. The higher bidder pays his brother the amount of the bid and gets the car. If bids are equal, ownership is determined by the toss of a coin and no money is exchanged. The first brother has \$500 at his disposal whereas the other has \$800.

Draw the game matrix. Is this a zero-sum game? How should the brothers bid?