Hennessy–Milner Logics

Weak and Branching Bisimulation

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Outline

- Review of Hennessy–Milner Logic
- A Hennessy–Milner Logic for Weak Bisimulation
- A Hennessy–Milner Logic for Branching Bisimulation
- Briefly, Rooted Weak and Branching Bisimulation
- Conclusion

Hennessy–Milner Logic

$$\varphi \coloneqq \langle a \rangle \varphi \mid \bigwedge \Phi \mid \neg \varphi \qquad (a \in A, \Phi \subseteq \Phi_{\mathrm{HML}})$$

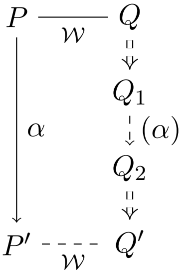
$$\begin{aligned} p \models \langle a \rangle \varphi &\iff \exists p'.\ (p \xrightarrow{a} p') \land p' \models \varphi \\ p \models \bigwedge \Phi &\iff \bigwedge \{ p \models \varphi \mid \varphi \in \Phi \} \\ p \models \neg \varphi &\iff p \not\models \varphi \end{aligned}$$

How to Deal With τ?

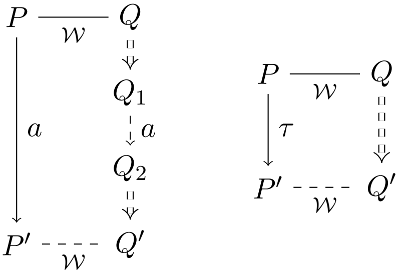
$$\varphi \coloneqq \langle \alpha \rangle \varphi \mid \langle \epsilon \rangle \varphi \mid \langle \hat{\tau} \rangle \varphi \mid \bigwedge \Phi \mid \neg \varphi \qquad (\alpha \in A_\tau, \Phi \subseteq \Phi_L)$$

$$\begin{aligned}
p \vDash \langle \alpha \rangle \varphi &\iff \exists p'.\ (p \xrightarrow{\alpha} p') \wedge p' \vDash \varphi \\
p \vDash \langle \epsilon \rangle \varphi &\iff \exists p'.\ (p \Rightarrow p') \wedge p' \vDash \varphi \\
p \vDash \langle \hat{\tau} \rangle \varphi &\iff \exists p'.\ (p = p' \vee (p \xrightarrow{\tau} p')) \wedge p' \vDash \varphi \\
p \vDash \bigwedge \Phi &\iff \bigwedge \{p \vDash \varphi \mid \varphi \in \Phi\} \\
p \vDash \neg \varphi &\iff p \nvDash \varphi
\end{aligned}$$

Weak Bisimulation



Weak Bisimulation



HML_w: Definition

Syntax

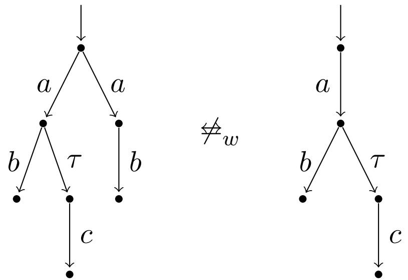
$$\varphi_w \coloneqq \langle\epsilon\rangle\langle a\rangle\langle\epsilon\rangle\varphi_w \mid \langle\epsilon\rangle\varphi_w \mid \bigwedge\Theta_w \mid \neg\varphi_w \qquad (a \in A, \Theta_w \subseteq \Phi_w)$$

Semantics

$$p \vDash \langle\epsilon\rangle\langle a\rangle\langle\epsilon\rangle\varphi \iff \exists p'.\ (p \Rightarrow\xrightarrow{a}\Rightarrow p') \wedge p' \vDash \varphi$$
$$\vdots$$

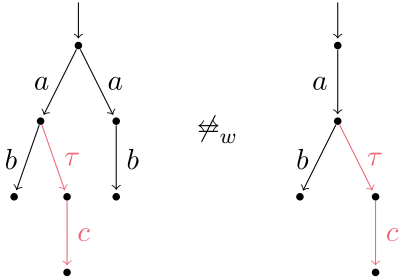
Also, abbreviate $\langle\epsilon\rangle\langle a\rangle\langle\epsilon\rangle\varphi$ to $\langle\epsilon a\epsilon\rangle\varphi$.
This logic characterises weak bisimulation.

HML_w: Example 1



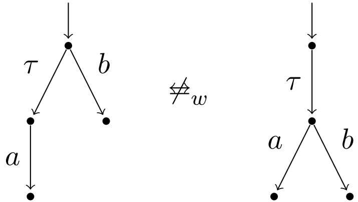
$$\langle \epsilon a \epsilon \rangle \neg \langle \epsilon c \epsilon \rangle \top$$

HML_w: Example 1



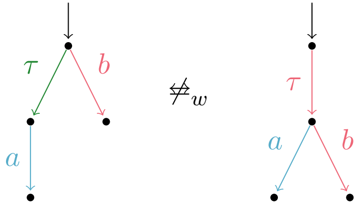
$$\langle \epsilon a \epsilon \rangle \neg \langle \epsilon c \epsilon \rangle \top$$

HML_w: Example 2



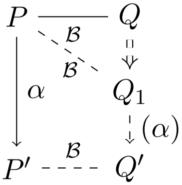
$$\langle\epsilon\rangle(\langle\epsilon a\epsilon\rangle\top \wedge \neg\langle\epsilon b\epsilon\rangle\top)$$

HML_w: Example 2

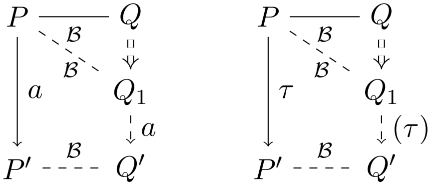


$$\langle \epsilon \rangle (\langle \epsilon a \epsilon \rangle \top \wedge \neg \langle \epsilon b \epsilon \rangle \top)$$

Branching Bisimulation



Branching Bisimulation



HML_b

$$\varphi_b \coloneqq \langle\epsilon\rangle(\varphi_b \wedge \langle a\rangle\varphi_b) \mid \langle\epsilon\rangle(\varphi_b \wedge \langle\hat{\tau}\rangle\varphi_b) \mid \bigwedge \Theta_b \mid \neg\varphi_b \qquad (a \in A, \Theta_b \subseteq \Phi_b)$$

Abbreviate

$$\varphi\langle a\rangle\psi \mathrel{\hat{=}} \varphi \wedge \langle a\rangle\psi$$
$$\varphi\langle\hat{\tau}\rangle\psi \mathrel{\hat{=}} \varphi \wedge \langle\hat{\tau}\rangle\psi$$

Then

HML_b

$$\varphi_b \coloneqq \langle\epsilon\rangle(\varphi_b\langle a\rangle\varphi_b) \mid \langle\epsilon\rangle(\varphi_b\langle\hat{\tau}\rangle\varphi_b) \mid \bigwedge \Theta_b \mid \neg\varphi_b \qquad (\Theta_b \subseteq \Phi_b)$$

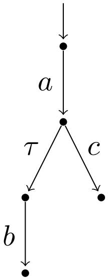
The idea is that $\varphi\langle a\rangle\psi$ captures testing 'just-before' an action.
Semantics

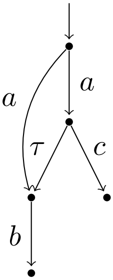
$$\begin{aligned} p \vDash \langle\epsilon\rangle(\varphi\langle a\rangle\psi) \quad &\Longleftrightarrow \quad \exists p'\, p''.\ (p \Rightarrow p' \xrightarrow{a} p'') \wedge p' \vDash \varphi \wedge p'' \vDash \psi \\ p \vDash \langle\epsilon\rangle(\varphi\langle\hat{\tau}\rangle\psi) \quad &\Longleftrightarrow \quad \exists p'\, p''.\ (p \Rightarrow p') \wedge (p' = p'' \vee (p' \xrightarrow{\tau} p'')) \wedge \\ &\qquad\qquad p' \vDash \varphi \wedge p'' \vDash \psi \end{aligned}$$

and note that

$$\begin{aligned} \langle\epsilon a\rangle\psi &\equiv \langle\epsilon\rangle(\top\langle a\rangle\psi) \\ \langle\epsilon\rangle\psi &\equiv \langle\epsilon\rangle(\top\langle\hat{\tau}\rangle\psi) \\ \langle\epsilon a\epsilon\rangle\psi &\equiv \langle\epsilon a\rangle\langle\epsilon\rangle\psi. \end{aligned}$$

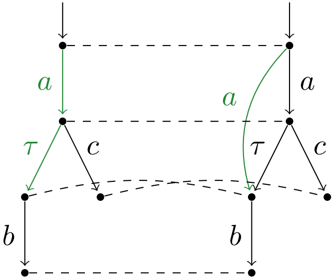
HML_b: Example 1





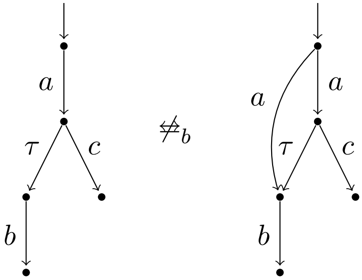
HML_b: Example 1

It is a weak bisimulation



HML_b: Example 1

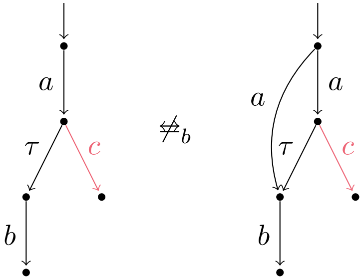
But it is not a branching bisimulation



$$\langle\epsilon\rangle(\top\langle a\rangle(\neg\langle\epsilon c\rangle))$$

HML_b: Example 1

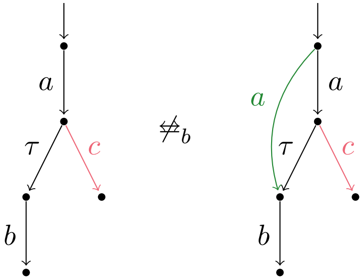
But it is not a branching bisimulation



$$\langle\epsilon\rangle(\top\langle a\rangle(\neg\langle\epsilon c\rangle\top))$$

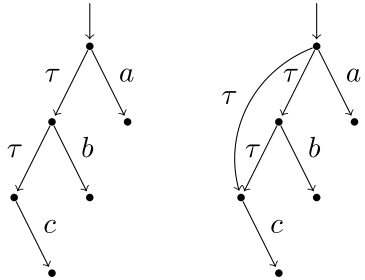
HML_b: Example 1

But it is not a branching bisimulation



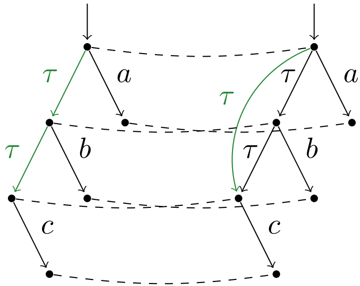
$$\langle\epsilon\rangle(\top\langle a\rangle(\neg\langle\epsilon c\rangle\top))$$

HML_b: Example 2



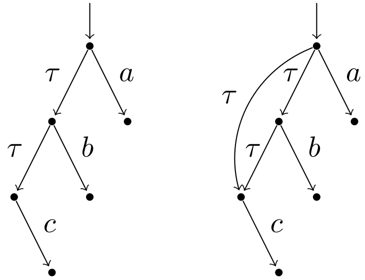
HML_b: Example 2

It is a weak bisimulation



HML_b: Example 2

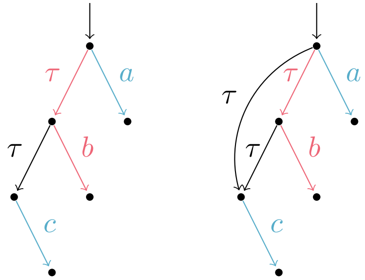
But it is not a branching bisimulation



$$\langle\epsilon\rangle\big((\langle\epsilon a\rangle\top)\langle\hat{\tau}\rangle(\langle\epsilon c\rangle\top \wedge \neg\langle\epsilon b\rangle\top)\big)$$

HML_b: Example 2

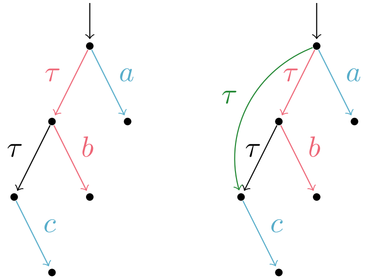
But it is not a branching bisimulation



$$\langle\epsilon\rangle\Big((\langle\epsilon a\rangle\top)\langle\hat{\tau}\rangle(\langle\epsilon c\rangle\top\wedge\neg\langle\epsilon b\rangle\top)\Big)$$

HML_b: Example 2

But it is not a branching bisimulation



$$\langle\epsilon\rangle\Big((\langle\epsilon a\rangle\top)\langle\hat{\tau}\rangle(\langle\epsilon c\rangle\top \wedge \neg\langle\epsilon b\rangle\top)\Big)$$

Rooted Bisimulations

Briefly,

$$\varphi_w \coloneqq \langle \epsilon a \epsilon \rangle \varphi_w \mid \langle \epsilon \rangle \varphi_w \mid \bigwedge \Theta_w \mid \neg \varphi_w \qquad (a \in A, \Theta_w \subseteq \Phi_w)$$

$$\varphi_{rw} \coloneqq \langle \epsilon \alpha \epsilon \rangle \varphi_w \mid \bigwedge \Theta_{rw} \mid \neg \varphi_{rw} \mid \varphi_w \qquad (\alpha \in A_\tau, \Theta_{rw} \subseteq \Phi_{rw})$$

$$\varphi_b \coloneqq \langle \epsilon \rangle (\varphi_b \langle a \rangle \varphi_b) \mid \langle \epsilon \rangle (\varphi_b \langle \hat{\tau} \rangle \varphi_b) \mid \bigwedge \Theta_b \mid \neg \varphi_b \qquad (a \in A, \Theta_b \subseteq \Phi_b)$$

$$\varphi_{rb} \coloneqq \langle \alpha \rangle \varphi_b \mid \bigwedge \Theta_{rb} \mid \neg \varphi_{rb} \mid \varphi_b \qquad (\alpha \in A_\tau, \Theta_{rb} \subseteq \Phi_{rb})$$

Sources & Further Reading

Algebraic Laws for Nondeterminism and Concurrency [Hennessy and Milner, 1980, 1985]: Introduced observational equivalence ($\approx$bisimulation) and HML. I recommend the 1985 version.

Linear Time—Branching Time Spectrum I and II (I: [van Glabbeek, 1990, 2001]; II: [van Glabbeek, 1993]): LtBtS-I presents a spectrum of equivalences between trace and bisimulation semantics; contains a nice discussion of Infinitary vs Non-Infinitary HML. LtBtS-II extends this to processes with τs, giving a modal characterisation of (among many others) weak and branching bisimulation.

Divide and Congruence I, II, III [Fokkink et al., 2012; Fokkink and van Glabbeek, 2016; Fokkink et al., 2019]: Decomposes the modalities of LtBtS-II into simple components; this is what we saw today.

Three Logics for Branching Bisimulation [De Nicola and Vaandrager, 1995]: Presents three modal logics for branching bisimulation, which are all different to the one I presented here.

Rocco De Nicola and Frits Vaandrager. 1995. Three Logics for Branching Bisimulation. J. ACM 42, 2 (March 1995), 458487. https://doi.org/10.1145/201019.201032

Wan Fokkink and Rob van Glabbeek. 2016. Divide and Congruence II: Delay and Weak Bisimilarity. In Proceedings of the 31st Annual ACM/IEEE Symposium on Logic in Computer Science (LICS 16). Association for Computing Machinery, New York, NY, USA, 778787. https://doi.org/10.1145/2933575.2933590

Wan Fokkink, Rob van Glabbeek, and Paulien de Wind. 2012. Divide and congruence: From decomposition of modal formulas to preservation of branching and -bisimilarity. Information and Computation 214 (2012), 59 – 85. https://doi.org/10.1016/j.ic.2011.10.011

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R. J. van Glabbeek. 1993. The linear time — Branching time spectrum II. In CONCUR'93, Eike Best (Ed.). Springer Berlin Heidelberg, Berlin, Heidelberg, 66–81. https://doi.org/10.1007/3-540-57208-2_6